HOW MANY TRIP REQUESTS COULD WE SUPPORT? AN ACTIVITY-TRAVEL BASED VEHICLE SCHEDULING APPROACH

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ABSTRACT

It is vital to have integrated model systems that fully capture the interactions between supply and demand dimensions of travel to model the implications of advanced technologies and mobility services on traveler behavior. In this research, we introduce a new state dimension (called the 'under-service trip request' state) to the vehicle scheduling model in order to track the execution status of the trip requests at any time and transportation node. We also construct activity-travel graphs for passengers to detect the execution of the passenger's activities. We further propose a time-discretized multi-commodity network flow model that not only guarantees that each activity request is systematically evaluated within its time window (depending on whether it is mandatory or optional), but also ensures that the road as well as vehicle capacity constraints are not violated. By introducing a mapping constraint between 'passenger's pickup/drop-off at an activity location' and 'under-service trip requests state in a vehicle network' as linking constraints, passenger and vehicle networks can be seamlessly connected together. By dualizing this set of trip request constraints and the road capacity constraints into the objective function and utilizing a Lagrangian relaxation approach, the main problem is decomposed to two sub-problems which can be solved in parallel through computationally efficient algorithms for real-world transportation networks. Based on a standard optimization solver and C++, we developed an open-source activity-based vehicle routing engine, namely Agent+, using real-world Phoenix subarea network data sets and trip requests generated from activity-based modelling system OpenAMOS.

Keywords: Time-dependent state-dependent shortest path problem, activity-based model, vehicle routing problem, dynamic network modeling

1. MOTIVATION AND BACKGROUND

The past decade has witnessed unprecedented advances in the auto industry, specifically in the domain of autonomous vehicle technologies. Several auto companies have forged new paths and introduced vehicles of the future that need minimal human intervention for their operation (Tesla Motors Team *(1)*; Sherman *(2)*). Ridesourcing, operated by Transportation Network Companies (TNCs) such as Uber and Lyft, is another game changing technology introduced in recent times. TNCs aim to provide reliable and inexpensive personalized travel options that combine the best of personalized transport (for example, door-to-door travel), as well as transit services (where the users pay per trip and do not have to drive the vehicle themselves). Recent reports show that 12% of registered voters across the United States used ridesourcing services at least once in the past month (Morning Consult *(3)*).

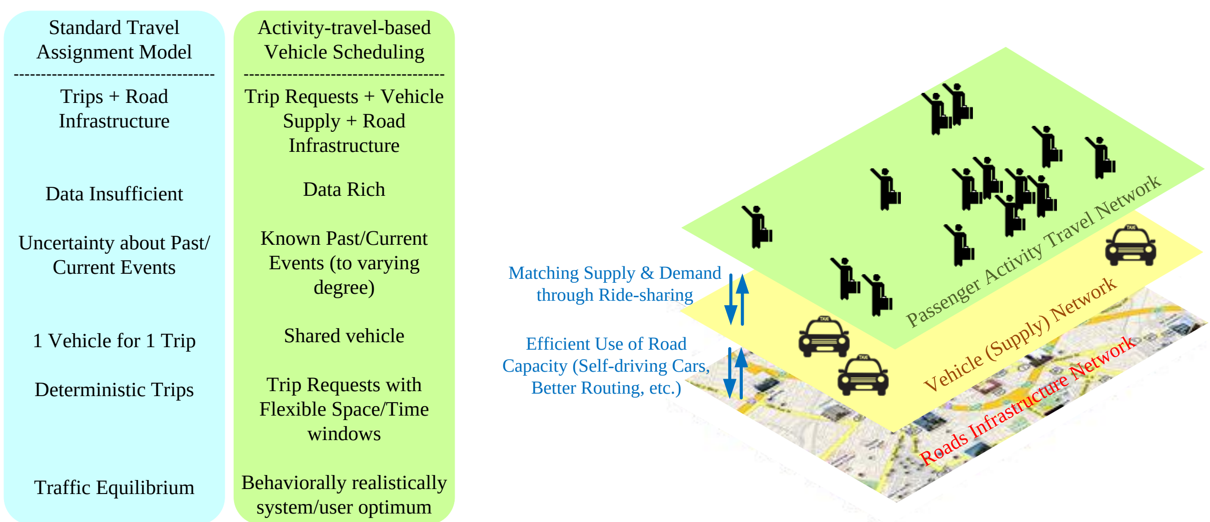
The rapidly growing popularity of TNCs coupled with autonomous vehicle technologies, could potentially redefine the way in which individuals schedule and execute their activities and also the way in which travel demand is managed by network operators. For the traveler, the freedom from having to drive could lead to more flexible activity schedules and increased productivity while travelling. On the other hand, network operators could handle demand by incentivizing/dis-incentivizing travel during a certain portion of the day (similar to surge pricing by Uber), or along a specific route. There is growing interest in the field to study incentive-based demand management strategies (for example, see Hu et al. *(4)*). It is therefore of critical importance to understand and accurately depict these transformative technologies and their implications for activity-travel patterns in travel demand model systems.

Great strides have been made in the past couple of decades in advancing travel demand modeling from the traditional 4-step travel demand models where demand and supply sides were considered static to present day state-of-the art integrated travel model systems. On the travel demand front, the profession has progressed from traditional trip-based methods to activity-based models (ABMs), which date back to the pioneering work of Kitamura *(5)* (see Rasouli and Timmermans *(6)* for a detailed synthesis on ABMs). ABMs view travel as derived demand, arising from the necessity of individuals to participate in various activities. This facilitates representing travel in a behaviorally realistic way in ABMs. On the other hand, network supply/simulation has progressed from static traffic assignment to dynamic traffic assignment (DTA) models that employ microscopic simulation and are capable of evaluating various traffic management strategies on the fly (*7*). There are ongoing efforts to tightly integrate the ABMs with DTA models with a view to accurately predict the impacts of dynamic pricing strategies and real-time information provision (Zockaie et al. *(8)*). While some of the integrated models operate in a sequential paradigm (exchange of information between the model components happens after a full iteration, for example Lin et al. *(9)*; Hao et al. *(10)*), others employ a tighter integration where information is exchanged on a more continuous basis (Balmer et al. *(11)*; Pendyala et al. *(12)*; Auld et al. *(13)*).

While the integrated models developed so far address modeling needs for the current array of travel options (modes, demand management strategies, etc.), they do not adequately handle emerging transportation technologies (ride-sharing services, autonomous vehicle technologies) that are increasingly penetrating the marketplace. For example, in an autonomous taxi fleet future, how would individuals go about scheduling their activities? How would the demand arising in such a situation impact network performance? Conversely, for a given fleet size, how many activities can a transportation networking company support? Integrated travel demand models of the present day would not be able to answer these questions for a variety of reasons.

ABMs still operate based on zonal level information (such as skims, by time-of-day) provided by DTA models. The ABMs are oblivious to network logistics such as availability of ridesourcing options and incentives/disincentives customized to specific trips/travelers. On the other hand, vehicle routing problems (VRP), used to depict ride-sharing services in DTA models, view travel as disjoint trips that are independent of each other (*14*). The solutions to VRP are typically optimization-based and lack a sound behavioral foundation. Solutions to VRP in the standard DTA models are often aimed at serving the maximum number of trip requests without taking into consideration the precedence constraints (or linkages) between the trips. Consider an individual's schedule comprising of three trips a) pick-up child, b) accompany child to the playground and c) take the child home. A VRP algorithm could produce a solution where trip requests for activities 'b' and 'c' are served, but in reality, activities 'b', and 'c' have a precedence constraint of engaging in activity 'a'. This vital behavioral constraint is ignored in the VRP optimization techniques incorporated in DTA models.

Figure 1(a) compares the characteristics of a standard dynamic traffic assignment system with the activity-travel based vehicle scheduling system. Due to the flexibility of the service offered by vehicle service providers and a vast variety of traveler' behaviors (e.g. different levels of traveler flexibility in terms of departure and/or arrival time windows, trip cost budget, and ride synchronization), future dynamic transportation network models must consist of various layers of passengers and vehicle service providers interacting with one another (Figure 1(b)).



(a) Comparison between standard DTA and new vehicle scheduling system

(b) Different layers of activity-travel vehicle scheduling system

FIGURE 1 (a) a comparison between standard traffic assignment and proposed activity-travel vehicle scheduling system (adapted from Mahmassani *(15)*); (b) layers of passengers' requests, vehicles, and roads infrastructure network .

The objective of this paper is two-fold i) add to the existing knowledge in the VRP domain by proposing an algorithm that incorporates behavioral realism into VRP; ii) facilitate the representation of emerging technologies in ABM-DTA integrated models by providing ABMs with a richer set of information made available by the proposed algorithm. The objective of the paper is achieved by formulating and solving a time-discretized multi-commodity network flow model. The solution for the proposed algorithm takes into account the time-space constraints as well as activity hierarchy (precedence) constraints. It is envisioned that the proposed algorithm would enable the provision of a much richer set of information to ABMs, thus enabling ABMs to include ride-sharing/ride-hailing as an additional mode of travel. For example, if individuals are provided with a price (in the form of a Lagrangian multiplier) to undertake a trip, they can determine whether or not to engage in a discretionary activity based on the prevailing 'price' for the trip to get to that activity.

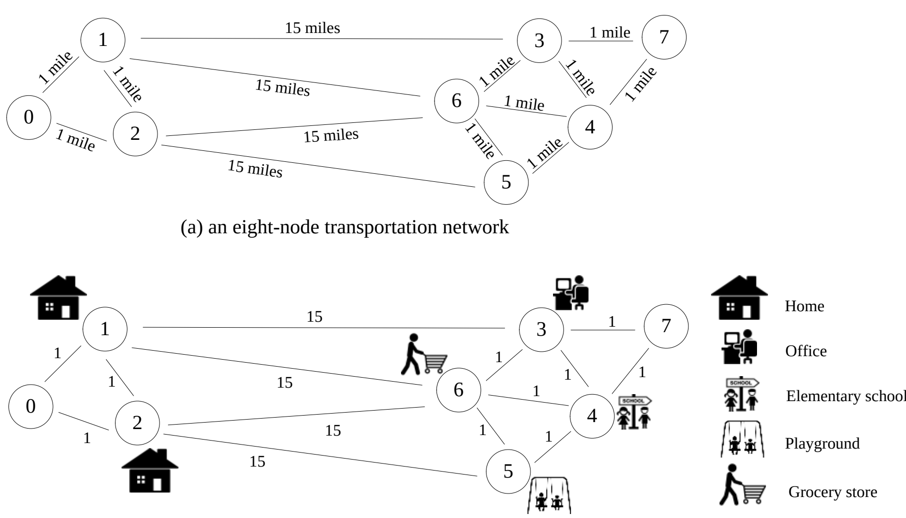
The remainder of the paper is organized as follows. The next section describes in detail, the construction of the activity-travel graphs for passengers and state-space-time networks for vehicles, and the third section presents the mathematical formulation of the time-discretized multi-commodity network flow model, as well as the solution approach. The fourth section provides results from the application of the proposed algorithm to the Phoenix subarea transportation network. Discussion, concluding remarks, and directions for future research form the fifth and final section of the paper.

2. NETWORK CONSTRUCTION FOR PASSENGERS AND VEHICLES

This section describes the construction of activity networks for passengers followed by the explanation of multi-dimensional state-space-time (SST) network construction for vehicles. In a vehicle network, adding the 'under-service trip requests state' dimension helps in tracking the execution status of the passengers' trip requests at any time. Interested readers are referred to a recent paper by Mahmoudi and Zhou *(16)* for more details about how to construct a state-space-time network.

2.1. Illustrative Example for Passenger Activity-travel Network Construction

This section details the construction of the graph containing passenger p's activities using an example. Take an eight-node transportation network, illustrated in Figure 2(a). Suppose two passengers, each with a specific origin and destination. Each passenger intends to perform a set of activities during the day (some of them are mandatory and others are optional). Table 1 presents the information related to the passengers' trip requests.



(a) an eight-node transportation network

(b) activities location on the transportation network

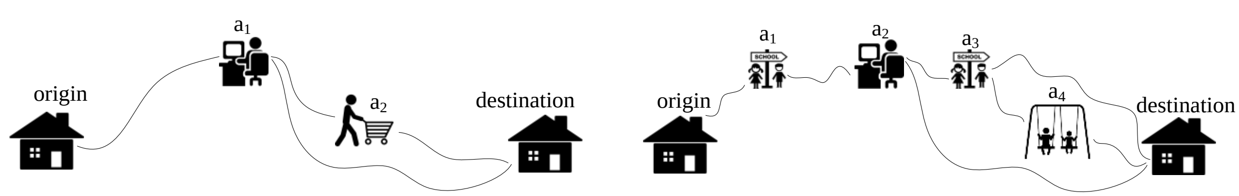
FIGURE 2 (a) an eight-node transportation network; (b) the transportation network in which the location of passengers' activities has been specified.

The location of passengers' activities has been depicted on the eight-node transportation network in Figure 2(b). In this example, passenger p_1 has a trip request from his home to office (working at office is a mandatory activity for p_1). Moreover, he would like to shop for groceries after work (note that this activity is optional). On the other hand, passenger p_2 has to drop off his kid at school first thing in the morning, and then go to work. Although both the activities in the morning are mandatory for passenger p_2, he has more flexible schedule (with discretionary activities) after work. He may (1) directly go home from office and assign the task of picking up the kid from school to his wife, (2) pick up the kid from school and go home, or (3) pick up the kid from school, take her to the playground and play with her for half an hour and then return home. In the latter alternative, taking the kid to the playground is dependent on picking her up from school.

TABLE 1 Information Related to the Trip Requests of Passengers

Passenger p_1			
Origin	**Location**		
Home	Node 1		
Destination	**Location**		
Home	Node 1		
Activity	**Location**	**Type**	**Time window**
Working at office	Node 3	Mandatory	[8:00 AM, 4:00 PM]
Shopping groceries	Node 6	Optional	[4:05 PM, 4:50 PM]
Passenger p_2			
Origin	**Location**		
Home	Node 2		
Destination	**Location**		
Home	Node 2		
Activity	**Location**	**Type**	**Time window**
Drop-off the kid at school	Node 4	Mandatory	[7:55 AM, 7:55 AM]
Working at office	Node 3	Mandatory	[8:00 AM, 4:00 PM]
Pick up the kid from school	Node 4	Optional	[4:10 PM, 4:10 PM]
Play with kid at playground	Node 5	Optional	[4:15 PM, 4:45 PM]

To construct the activity graph for each passenger, it is sufficient to arrange all possible trip requests respecting their type and time window. Figure 3(a) and 3(b) presents the graphs of travel activities for passengers p_1 and p_2, respectively. In these graphs, each passenger should start and end his activity trip chain (or a tour) at the same location (home).

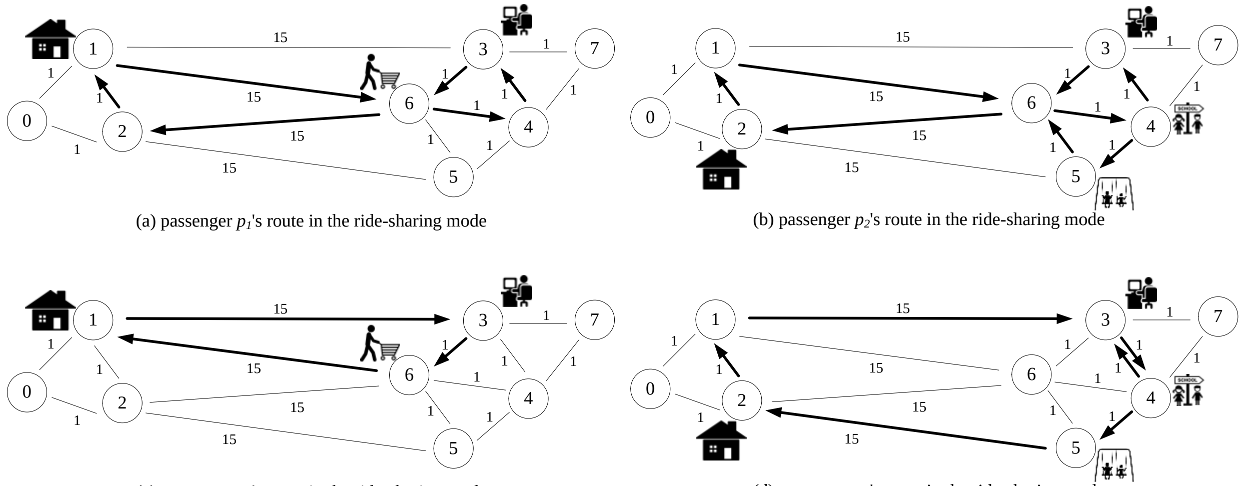


(a) passenger p_1's activity network (b) passenger p_2's activity network

FIGURE 3 (a) passenger p_1's activity-travel graph; (b) passenger p_2's activity-travel graph.

2.2. Vehicle State-space-time Network Construction

The construction of multi-dimensional state-space-time (SST) networks for vehicles is explained in this section with the help of the example mentioned above. Note that in the SST network representation, an activity can only performed along the corresponding activity link. In order to consider the precedence constraints (e.g. drop-off a passenger at an activity location should occur before his pickup from there) as well as vehicle capacity constraints, a new dimension called as the "under-service trip requests state" is introduced. With the help of this definition, the execution status of passengers' trip requests in each vehicle can be tracked at any time within the vehicle time horizon. In the example mentioned above, we assume that the vehicle has 3 seats available for serving the passengers. Let $r(p, a, a')$ denote passenger p's trip request in which he leaves activity location a to perform activity a'. From Figure 3(a), there might be four trip requests for passenger p_1, i.e. $r_{p_1}^1 = r(p_1, origin, a_1)$, $r_{p_1}^2 = r(p_1, a_1, a_2)$, $r_{p_1}^3 = r(p_1, a_1, destination)$, $r_{p_1}^4 = r(p_1, a_2, destination)$, while according to Figure 3(b), seven trip requests can be possible for passenger p_2, i.e. $r_{p_2}^1 = r(p_2, origin, a_1)$, $r_{p_2}^2 = r(p_2, a_1, a_2)$, $r_{p_2}^3 = r(p_2, a_2, destination)$, $r_{p_2}^4 = r(p_2, a_2, a_3)$, $r_{p_2}^5 = r(p_2, a_3, destination)$, $r_{p_2}^6 = r(p_2, a_3, a_4)$, $r_{p_2}^7 = r(p_2, a_4, destination)$. The under-service trip requests state (denoted by w from now on) is explained with the help of Table 2. Table 2 presents the under-service trip requests state w at any node i and time t. Note that $w = [\_\_\_]$ is the null state in which the vehicle is not involved in any passenger's trip request. Figures 4(a) and 4(b) show the route of the passengers when they share their ride with each other. Figure 5(a) also depicts the vehicles two-dimensional space-time network when two passengers are served through the ride-sharing.



(a) passenger p_1's route in the ride-sharing mode
(b) passenger p_2's route in the ride-sharing mode
(c) passenger p_1's route in the ride-sharing mode
(d) passenger p_2's route in the ride-sharing mode

FIGURE 4 (a) passenger p_1's route in the ride-sharing mode; (b) passenger p_2's route in the ride-sharing mode; (c) passenger p_1's route if he drives alone; (d) passenger p_2's route if he drives alone.

TABLE 2 State Transitions and Trips for the Aforementioned Example

Trips in the morning					
from node i	**at time t**	**at state w**	**to node i'**	**at time t'**	**at state w'**
Node 0	7:20 AM	$[\_\ \_\ \_]$	Node 2	7:25 AM	$[\_\ \_\ \_]$
Node 2	7:25 AM	$[\_\ \_\ \_]$	Node 2	7:25 AM	$[r^1_{p_2}\ r^1_{p_2}\ \_]$
Node 2	7:25 AM	$[r^1_{p_2}\ r^1_{p_2}\ \_]$	Node 1	7:30 AM	$[r^1_{p_2}\ r^1_{p_2}\ \_]$
Node 1	7:30 AM	$[r^1_{p_2}\ r^1_{p_2}\ \_]$	Node 1	7:30 AM	$[r^1_{p_2}\ r^1_{p_2}\ r^1_{p_1}]$
Node 1	7:30 AM	$[r^1_{p_2}\ r^1_{p_2}\ r^1_{p_1}]$	Node 6	7:50 AM	$[r^1_{p_2}\ r^1_{p_2}\ r^1_{p_1}]$
Node 6	7:50 AM	$[r^1_{p_2}\ r^1_{p_2}\ r^1_{p_1}]$	Node 4	7:55 AM	$[r^1_{p_2}\ r^1_{p_2}\ r^1_{p_1}]$
Node 4	7:55 AM	$[r^1_{p_2}\ r^1_{p_2}\ r^1_{p_1}]$	Node 4	7:55 AM	$[\_\ r^2_{p_2}\ r^1_{p_1}]$
Node 4	7:55 AM	$[\_\ r^2_{p_2}\ r^1_{p_1}]$	Node 3	8:00 AM	$[\_\ r^2_{p_2}\ r^1_{p_1}]$
Node 3	8:00 AM	$[\_\ r^2_{p_2}\ r^1_{p_1}]$	Node 3	8:00 AM	$[\_\ \_\ \_]$
Node 3	8:00 AM	$[\_\ \_\ \_]$	Node 7	8:05 AM	$[\_\ \_\ \_]$
Trips in the afternoon					
from node i	**at time t**	**at state w**	**to node i'**	**at time t'**	**at state w'**
Node 7	3:55 PM	$[\_\ \_\ \_]$	Node 3	4:00 PM	$[\_\ \_\ \_]$
Node 3	4:00 PM	$[\_\ \_\ \_]$	Node 3	4:00 PM	$[r^4_{p_2}\ r^2_{p_1}\ \_]$
Node 3	4:00 PM	$[r^4_{p_2}\ r^2_{p_1}\ \_]$	Node 6	4:05 PM	$[r^4_{p_2}\ r^2_{p_1}\ \_]$
Node 6	4:05 PM	$[r^4_{p_2}\ r^2_{p_1}\ \_]$	Node 6	4:05 PM	$[r^4_{p_2}\ \_\ \_]$
Node 6	4:05 PM	$[r^4_{p_2}\ \_\ \_]$	Node 4	4:10 PM	$[r^4_{p_2}\ \_\ \_]$
Node 4	4:10 PM	$[r^4_{p_2}\ \_\ \_]$	Node 4	4:10 PM	$[r^6_{p_2}\ r^6_{p_2}\ \_]$
Node 4	4:10 PM	$[r^6_{p_2}\ r^6_{p_2}\ \_]$	Node 5	4:15 PM	$[r^6_{p_2}\ r^6_{p_2}\ \_]$
Node 5	4:15 PM	$[r^6_{p_2}\ r^6_{p_2}\ \_]$	Node 5	4:15 PM	$[\_\ \_\ \_]$
Node 5	4:15 PM	$[\_\ \_\ \_]$	Node 5	4:45 PM	$[\_\ \_\ \_]$
Node 5	4:45 PM	$[\_\ \_\ \_]$	Node 5	4:45 PM	$[r^7_{p_2}\ r^7_{p_2}\ \_]$
Node 5	4:45 PM	$[r^7_{p_2}\ r^7_{p_2}\ \_]$	Node 6	4:50 PM	$[r^7_{p_2}\ r^7_{p_2}\ \_]$
Node 6	4:50 PM	$[r^7_{p_2}\ r^7_{p_2}\ \_]$	Node 6	4:50 PM	$[r^7_{p_2}\ r^7_{p_2}\ r^4_{p_1}]$
Node 6	4:50 PM	$[r^7_{p_2}\ r^7_{p_2}\ r^4_{p_1}]$	Node 2	5:10 PM	$[r^7_{p_2}\ r^7_{p_2}\ r^4_{p_1}]$
Node 2	5:10 PM	$[r^7_{p_2}\ r^7_{p_2}\ r^4_{p_1}]$	Node 2	5:10 PM	$[\_\ \_\ r^4_{p_1}]$
Node 2	5:10 PM	$[\_\ \_\ r^4_{p_1}]$	Node 1	5:15 PM	$[\_\ \_\ r^4_{p_1}]$
Node 1	5:15 PM	$[\_\ \_\ r^4_{p_1}]$	Node 1	5:15 PM	$[\_\ \_\ \_]$
Node 1	5:15 PM	$[\_\ \_\ \_]$	Node 0	5:20 PM	$[\_\ \_\ \_]$

By using the ride-sharing mode for the passengers' trip requests, several miles are deducted. Figures 4(c) and 4(d) illustrate the route of the passengers and Figures 5(b) and 5(c) depict the corresponding mileage for performing their respective activities by using their own vehicle. In these figures, passenger p_1 travels for 31 miles, whereas p_2 travels for 34 miles. Therefore, if each passenger drives separately, they spend 65 miles in total to perform their activities, while according to Figure 5(a), if they share a ride with each other (for a portion of their tours), the vehicle only travels for 43 miles which is 22 miles less than driving alone.

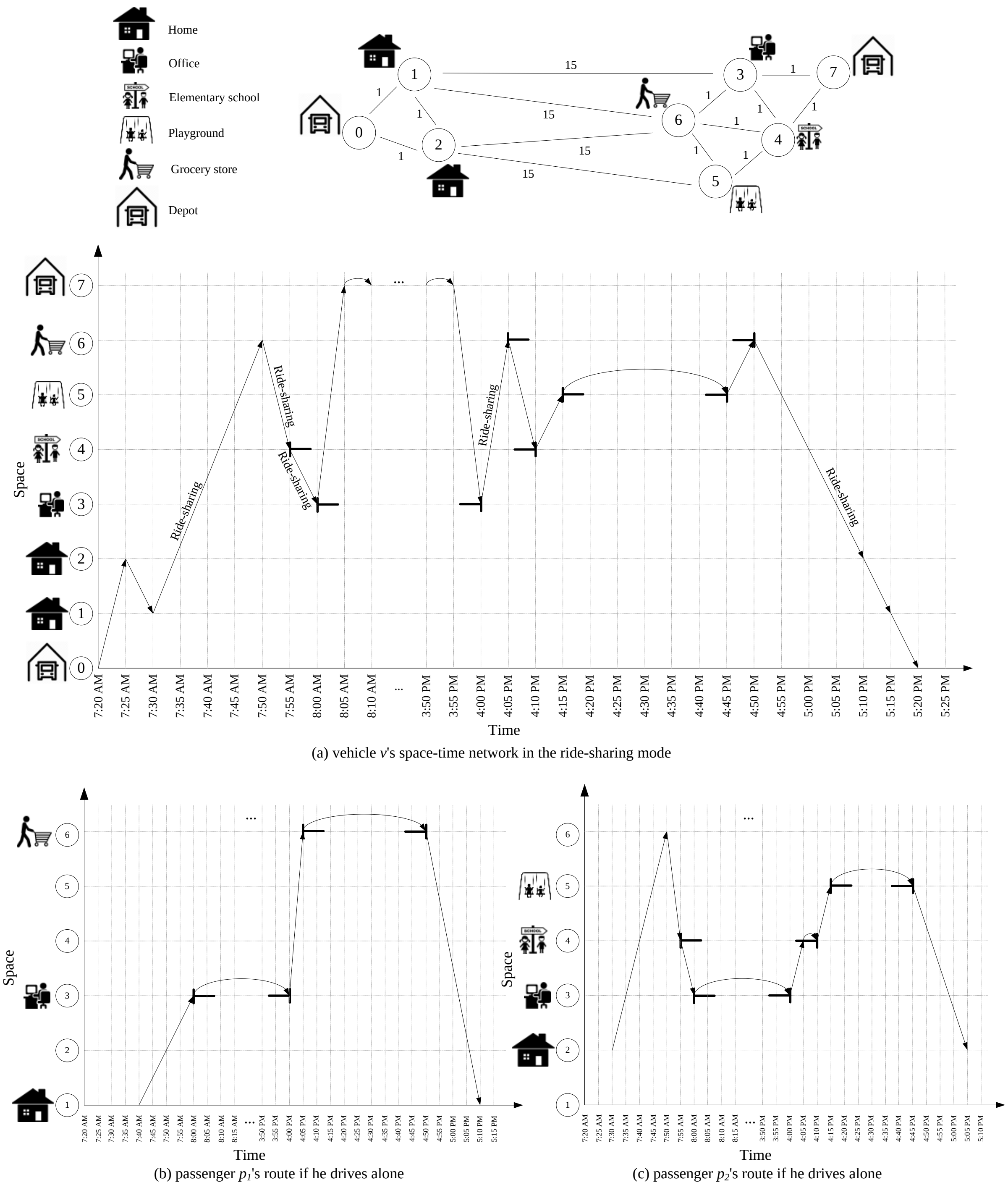


FIGURE 5 (a) vehicle space-time network in the ride-sharing mode; (b) passenger p_1's route when he drives by his own car; (c) passenger p_2's route when he drives by his own car.

3. TIME-DISCRETIZED MULTI-COMMODITY NETWORK FLOW PROGRAMMING MODEL

In this section, we initially present the mathematical programming of the proposed time-discretized multi-commodity network flow model. We further apply Lagrangian Relaxation (LR) approach to relax the two groups of complicated constraints into the objective function. By doing so, the main problem is systematically decomposed to two sub-problems where sub-problem (1) is a typical least cost path problem and sub-problem (2) is a time-dependent state-dependent least cost path problem. Both sub-problems can be solved by computationally efficient algorithms for solving the shortest path problem, e.g. dynamic programming, label correcting algorithm, etc.

3.1. Mathematical Model

Mathematical formulation for the proposed time-discretized multi-commodity network flow model is presented in this section. This formulation not only guarantees that each activity (depending on whether it is mandatory or optional) is performed within its time window, but also ensures that the road as well as vehicle capacity constraints are not violated. Note that the roads' capacity constraints can be constructed based on the cumulative arrival and departure functions to reflect detailed traffic congestion propagation through simplified kinematic wave model (*17*). In this model, we assume that all vehicles have the same planning horizon, i.e. $[0, T]$. Moreover, to distinguish regular transportation nodes from passengers' origin, destination, activities location, as well as vehicles' origin and destination, we add dummy nodes corresponding each to the original transportation network. Each dummy node is only connected to its corresponding transportation node by a link. The travel time on this link can be interpreted as the service time if the added dummy node is related to a passenger's pickup/drop-off, and as preparation time if it is related to a vehicle's departure/arrival at a depot. Without loss of generality, this travel time is assumed to be a unit of time for all passengers and vehicles in this paper. Table 3 lists the notations for the sets, indices, parameters, and variables in this model.

TABLE 3 Sets, Indexes, Parameters, and Variables Used in Our Proposed Model

Symbol	Definition
k	Vehicle index
p	Passenger index
w, w'	Under-service trip requests state indices in vehicles networks
(i, i')	Index of a physical link between adjacent nodes i and i'
$TT(i, i', t)$	Link travel time from node i to node i' starting at time t
(i, t, w), (i', t', w')	Indexes of state-space-time vertices for vehicles SST network
(i, i', t, t', w, w')	Index of a space-time-state arc indicating vehicle v travels from node i at time t with under-service trip requests state w to node i' at time t' with under-service trip requests state w'
a, a'	Passengers' activity indices in passenger p's activities graph
$r(p, a, a')$	Trip request in which passenger p leaves activity location a to perform activity a'
$u(p, a, a')$	Utility gained from serving trip request $r(p, a, a')$
$\mu(i, i', t)$	Maximum road capacity per unit time interval on physical link (i, i') at time t
$\phi(p, a, a', i, i', t, t+1)$	$= 1$, if (i, t) is the dummy vertex from which passenger p calls a vehicle to be picked up for trip request $r(p, a, a')$ (trip $r(p, a, a')$ starts); $= 0$ otherwise
$\psi(p, a, a', i, i', t-1, t)$	$= 1$, if (i', t) is the dummy vertex at which passenger p is dropped off exactly when trip request $r(p, a, a')$ is completed; $= 0$ otherwise
$\Omega(p, a, a', i, t)$	Set of all feasible arcs from dummy vertex (i, t, w) to (i', t, w') in which state w' contains $r(p, a, a')$, while w does not (pickup).
$\Theta(p, a, a', i', t)$	Set of all feasible arcs from $(i, t-1, w)$ to dummy vertex (i', t, w') in which state w contains $r(p, a, a')$, while w' does not (drop-off).
$y(k, i, i', t, t', w, w')$	$= 1$ if arc (i, i', t, t', w, w') is used by vehicle k; $= 0$ otherwise
$x(p, a, a')$	$= 1$ if link (a, a') is traversed by passenger p; $= 0$ otherwise

Note that each vehicle k must start its route from the dummy node corresponding to its origin depot at time $t = 0$ with the null state. We call this vertex as super source vertex(k). In addition, vehicle k must ends its route at the dummy node corresponding to its destination depot at time $t = T$ with the null state. This vertex is called as super sink vertex(k). Finally, this time-discretized multi-commodity network flow problem can be formulated as follows:

$$Max \sum_{p,a,a'}[u(p,a,a').x(p,a,a')] \tag{1}$$

$s.t.$

— **[Passenger Activity Network Flow Balance]** Flow balance constraint at any node belongs to passenger p's activity network

$$\sum_{a'} x(p,a,a') - \sum_{a'} x(p,a',a) = b \tag{2}$$

$b = +1$, if a: passenger p's origin; $b = -1$, if a: passenger p's destination; $b = 0$; otherwise.

— **[Vehicle Network Flow Balance]** Flow balance constraint at any vertex belongs to vehicle k's SST network

$$\sum_{i',t',w'} y(k,i,i',t,t',w,w') - \sum_{i',t',w'} y(k,i',i,t',t,w',w) = b' \tag{3}$$

$b' = +1$, if (i,t,w): super source vertex(k); $b' = -1$, if (i,t,w): super sink vertex(k); $b' = 0$, otherwise.

— **[Pickup]** Coupling constraint to link 'the execution of passenger p's pickup for the trip request $r(p,a,a')$' to 'corresponding pickup arc in vehicle k's SST network'

$$\sum_{k,(w,w')\in\Omega(p,a,a',i,t)} y(k,i,i',t,t+1,w,w') = \phi(p,a,a',i,i',t,t+1).x(p,a,a') \qquad \forall\phi > 0 \tag{4}$$

— **[Delivery]** Coupling constraint to link 'the execution of passenger p's drop-off exactly when trip request $r(p,a,a')$ is completed' to 'corresponding delivery arc in vehicle k's SST network'

$$\sum_{k,(w,w')\in\Theta(p,a,a',i',t)} y(k,i,i',t-1,t,w,w') = \psi(p,a,a',i,i',t-1,t).x(p,a,a') \qquad \forall\psi > 0 \tag{5}$$

— **[Road Capacity]** Conceptual road outflow capacity constraint

$$\sum_k \sum_{t',w,w'} y(k,i,i',t,t',w,w') \le \mu(i,i',t) \qquad \forall(i,i');\ t \in [0,T-1] \tag{6}$$

— **[Binary Variables]** Binary definitional constraint

$$x(p,a,a') \in \{0,1\} \qquad \forall p,a,a' \tag{7}$$

$$y(k,i,i',t,t',w,w') \in \{0,1\} \qquad \forall k,i,i',t,t',w,w' \tag{8}$$

3.2.LAGRANGIAN RELAXATION-BASED SOLUTION APPROACH

Defining multi-dimensional decision variables $y(k,i',i',t',t',w,w')$ leads to computational challenges for the real-world data sets, which are addressed properly by specialized programs and an innovative solution framework. We reformulate the problem by relaxing the complex set of constraints (4), (5), and (6) into the objective function and introducing the corresponding Lagrangian multipliers, $\alpha(p,a,a')$, $\beta(p,a,a')$, and $\gamma(i,i',t)$ to construct the dualized Lagrangian function (9).

$$L = Min \sum_{p,a,a'}[-u(p,a,a').x(p,a,a')] + \sum_{p,a,a'} \alpha(p,a,a').[\sum_{k,(w,w')\in\Omega(p,a,a',i,t)} y(k,i,i',t,t+1,w,w') - \phi(p,a,a',i,i',t,t+1).x(p,a,a')] + \sum_{p,a,a'} \beta(p,a,a').[\sum_{k,(w,w')\in\Theta(p,a,a',i',t)} y(k,i,i',t-1,t,w,w') - \psi(p,a,a',i,i',t-1,t).x(p,a,a')] + \sum_{i,i',t} \gamma(i,i',t).\left[\sum_k \sum_{t',w,w'} y(k,i',i',t',t',w,w') - \mu(i,i',t)\right] \quad (9)$$

Based on a Lagrangian reformulation framework, the main problem can be transformed to two easy sub-problems, P_x and P_y, which can be solved independently with much computationally efficient effort (*15*).

Sub-problem P_x (10)
$Min\ (-U - A\Phi - B\Psi)X$
s.t.
Constraints (2) & (7)

Sub-problem P_y (11)
$Min\ (A + B + \Gamma)Y - \Gamma M$
s.t.
Constraints (3) & (8)

Sub-problem P_x is a typical least cost path problem, and P_y is a time-dependent state-dependent least cost path problem. Both sub-problems can be solved by computationally efficient algorithms, e.g. dynamic programming, label correcting algorithm, etc. In this research, we apply time-dependent state-dependent forward dynamic programming to solve these two sub-problems.

If individuals are provided with a price (in the form of lagrangian multipliers α and β) to undertake a trip, they can determine whether or not to engage in a discretionary activity based on the prevailing ‘price’ for the trip to get to that activity. In Section 4, the results from the application of the proposed algorithm on Phoenix subarea transportation network are provided.

4. COMPUTATIONAL EXPERIMENTS

The time-dependent state-dependent forward dynamic programming described in this paper is coded in C++ platforms. The experiments were performed on an Intel Workstation running two Xeon E5-2680 processors clocked at 2.80 GHz with 20 cores and 192 GB RAM running Windows Server 2008 x64 Edition. In addition, parallel computing and OpenMP technique are implemented for generating lower bound and upper bound at each iteration in the Lagrangian relaxation algorithm.

In this section, we examine our proposed model on sample data sets from the Phoenix subarea with 1162 transportation nodes and 3164 links, illustrated in Figure (6), to demonstrate the computational efficiency, as well as, solution optimality of the proposed algorithm. Some sample trip requests (an OD pairs) are illustrated by directed dashed links, whereas vehicles’ paths are shown by directed thick links with different colors from transportation links color.

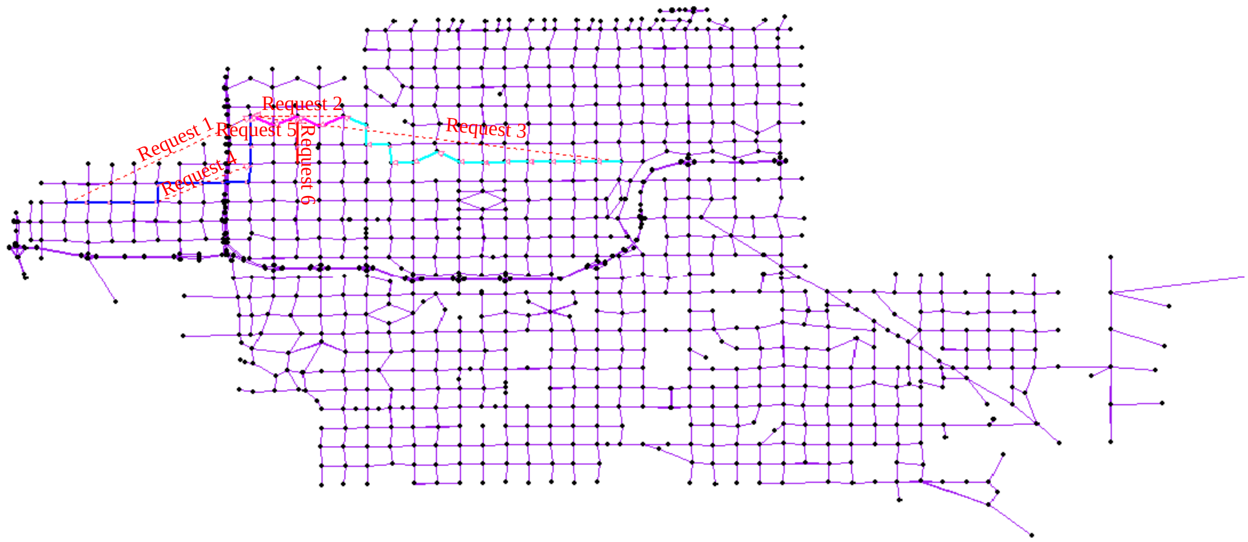


FIGURE 6 Phoenix Metropolitan Chandler subarea transportation network with 6 trip requests.

It is assumed that the routing cost of a transportation arc is $22/h, while the waiting cost at a node is $15/h. The initial charge is assumed to be $7 for all passengers. The maximum capacity of the vehicles for service is 2 seats, and the length of time horizon is 2 hours (120 min). It is also assumed that a unit of time has 1 min length. The proposed multi-commodity flow programming model is first demonstrated by the general purpose optimization package GAMS (Rosenthal *(18)*) in small transportation networks. For large-scale applications, we also create a time-dependent state-dependent shortest path computational engine by enhancing an open-source mesoscopic dynamic traffic assignment model namely DTALite (Zhou and Taylor *(19)*). The resulting open-source project with GAMS and C++ source codes can be found at https://github.com/xzhou99/Agent-Plus.

Trip requests on the test network are generated from an open-source activity based travel demand modeling system OpenAMOS (*12*). We also pre-specify the locations of vehicle depots at major activity locations in the test network. Table 4 presents the summary of vehicle routing results for a few test cases, with the following observations. If the number of trip requests increases, we generally need more vehicles to satisfy the travel demand desires. The ratio of required fleet size and total request number dramatically varies, between about 20% and 66%, from depending on the underlying spatial and temporal patterns. Typically, a larger pool of travel requests could lead to better system vehicle use efficiency. Due to the fixed vehicle depot and time-window restrictions, there are still a few under-served trip requests in the given passenger activity-travel pattern. Different from commonly used heuristic algorithms, the developed algorithm aims to find the exact or close-to-optimal solution to the proposed optimization model. The desirable trip-to-vehicle assignment and detailed routing solution could take about 5 min to compute for a medium case with about 50 trip requests.

TABLE 4 Results for the Phoenix Subarea with 1162 Nodes and 3164 Links

Test case	Number of trip requests	Number of vehicles required	Number of passengers not served	Running time (s)
1	6	4	0	8.41
2	10	5	0	25.5
3	15	10	1	51.7
4	37	20	1	255.9
5	48	9	0	308.2

We also explain the pricing mechanism by test case 1 with 6 trip requests. Figure 7 demonstrates the Lagrangian multiplier corresponding each trip request along 5 iterations. As we can see in Figure 7, each multiplier ultimately converges to a specific value. This value can be literally interpreted as the 'ultimate price' of a trip to get to a specific travel activity. Through this pricing mechanism, vehicle service providers would be able to offer a reasonable bid to their customers.

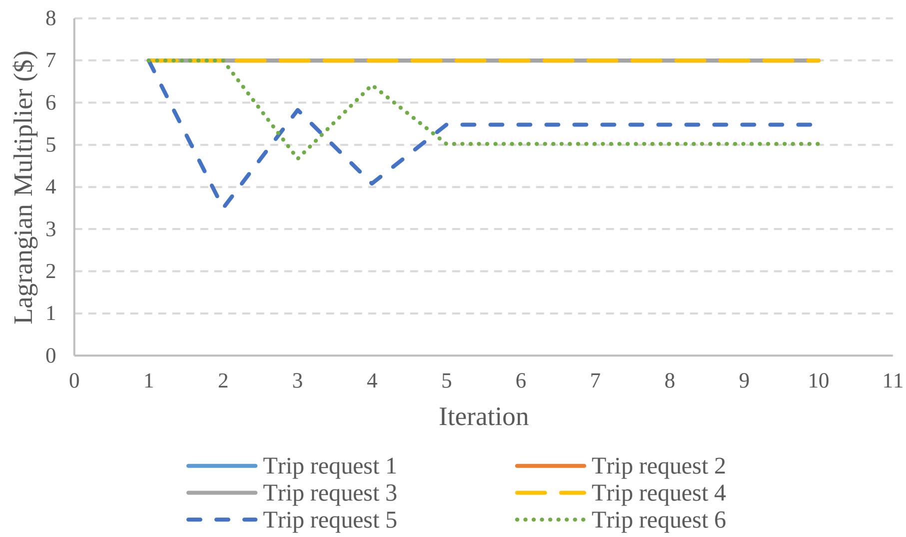


FIGURE 7 Lagrangian multipliers along 10 iterations in test case 1.

5. CONCLUSIONS

Despite all advancements in the real-time traffic control, DTA modelers still seek for a robust framework to extend their existing model (1) from single-OD demand to trip chaining, and (2) from driving your own mode to shared-use vehicle systems. In this research, it is expected that with the help of the proposed algorithm, it would be possible to provide a much richer set of information to ABMs, thus enabling ABMs to include ride-sharing/ride-hailing as an additional mode of travel.

Future research directions include (1) how to incorporate different activity-travel behavior decision-making rules to enhance the relatively simple utility-maximization objective function, (2) how to schedule activity-travel requests at extremely large scales to meet temporally and spatially distributed traveler demand, and (3) how to seamlessly integrate distributed computing, car platooning, and resource-oriented pricing and scheduling for better coordinated use of vehicles and road infrastructure resources. We hope this research line could offer a set of novel techniques on holistic behaviorally oriented traveler mobility optimization under the new environment of shared self-driving car networks.

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