

Stochastic Frontier Estimation of Budgets for Kuhn-Tucker Demand Systems: Application to Activity Time-use Analysis

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ABSTRACT

We propose a stochastic frontier approach to estimate budgets for the multiple discrete-continuous extreme value (MDCEV) model. The approach is useful when the underlying time and/or money budgets driving a choice situation are unobserved, but the expenditures on the choice alternatives of interest are observed. Several MDCEV applications hitherto used the observed total expenditure on the choice alternatives as the budget to model expenditure allocation among choice alternatives. This does not allow for increases or decreases in the total expenditure due to changes in choice alternative-specific attributes, but only allows a reallocation of the observed total expenditure among different alternatives. The stochastic frontier approach helps address this issue by invoking the notion that consumers operate under latent budgets that can be conceived (and modeled) as the maximum possible expenditure they are willing to incur. The proposed method is applied to analyze the daily out-of-home activity participation and time-use patterns in a survey sample of non-working adults in Florida. First, a stochastic frontier regression is performed on the observed out-of-home activity time *expenditure* (OH-ATE) to estimate the unobserved out-of-home activity time *frontier* (OH-ATF). The estimated *frontier* is interpreted as a subjective limit or maximum possible time individuals can allocate to out-of-home activities and used as the time budget governing out-of-home time-use choices in an MDCEV model. The efficacy of this approach is compared with other approaches for estimating time budgets for the MDCEV model, including: (a) a log-linear regression on the total observed expenditure for out-of-home activities, and (b) arbitrarily assumed, constant time budgets for all individuals in the sample. A comparison of predictive accuracy in time-use patterns suggests that the stochastic frontier and log-linear regression approaches perform better than arbitrary assumptions on time budgets. Between the stochastic frontier and log-linear regression approaches, the former results in slightly better predictions of activity participation rates while the latter results in slightly better predictions of activity durations. A comparison of policy simulations demonstrates that the stochastic frontier approach allows for the total out-of-home activity time expenditure to either expand or shrink due to changes in alternative-specific attributes. The log-linear regression approach allows for changes in total time expenditure due to changes in decision-maker attributes, but not due to changes in alternative-specific attributes.

1 INTRODUCTION

Numerous consumer choices are characterized by “multiple discreteness” where consumers can potentially choose multiple alternatives from a set of discrete alternatives available to them. Along with such discrete-choice decisions of which alternative(s) to choose, consumers typically make continuous-quantity decisions on how much of each chosen alternative to consume. Such multiple discrete-continuous (MDC) choices are being increasingly recognized and analyzed in a variety of social sciences, including transportation, economics, and marketing.

A variety of approaches have been used to model MDC choices. Among these, an increasingly popular approach is based on the classical microeconomic consumer theory of utility maximization. Specifically, consumers are assumed to optimize a direct utility function $U(\mathbf{t})$ over a set of non-negative consumption quantities $\mathbf{t} = (t_1, \dots, t_k, \dots, t_K)$ subject to a budget constraint, as below:

$$\text{Max } U(\mathbf{t}) \text{ such that } \sum_{k=1}^K p_k t_k = y \text{ and } t_k \geq 0 \forall k = 1, 2, \dots, K \quad (1)$$

In the above Equation, $U(\mathbf{t})$ is a quasi-concave, increasing, and continuously differentiable utility function of the consumption quantities, p_k ($k = 1, 2, \dots, K$) are unit prices for all goods, and y is a budget for total expenditure. A particularly attractive approach for deriving the demand functions from the utility maximization problem in Equation (1), due to Hanemann (1978) and Wales and Woodland (1983), is based on the application of Karush-Kuhn-Tucker (KT) conditions of optimality with respect to the consumption quantities. When the utility function is assumed to be randomly distributed over the population, the KT conditions become randomly distributed and form the basis for deriving the probability expressions for consumption patterns. Due to the central role played by the KT conditions, this approach is called the KT demand systems approach (or KT approach, in short).

Over the past decade, the KT approach has received significant attention for the analysis of MDC choices in a variety of fields, including environmental economics (von Haefen and Phaneuf, 2005), marketing (Kim et al., 2002), and transportation. In the transportation field, the multiple discrete-continuous extreme value (MDCEV) model formulated by Bhat (2005, 2008) has lead to an increased use of the KT approach for analyzing a variety of choices, including individuals’ activity participation and time-use (Habib and Miller, 2008; Pinjari et al., 2009; Chikaraishi et al., 2010; Eluru et al., 2010; Spissu et al., 2011; Sikder and Pinjari, 2014),

household vehicle ownership and usage (Ahn et al., 2008; Jaggi et al., 2011; Sobhani et al., 2013; Faghieh-Imani et al., 2014), recreational/leisure travel choices (von Haefen and Phaneuf, 2005; Van Nostrand et al., 2013), energy consumption choices, and builders' land-development choices (Farooq et al., 2013; Kaza et al., 2010). Thanks to these advances, KT-based MDC models are being increasingly used in empirical research and have begun to be employed in operational travel forecasting models (Bhat et al., 2013a). On the methodological front, recent literature in this area has started to enhance the basic formulation in Equation (1) along three specific directions: (a) toward more flexible, non-additively separable utility functions that accommodate rich substitution and complementarity patterns in consumption (Bhat et al., 2013b), (b) toward more flexible stochastic specifications for the random utility functions (Pinjari and Bhat, 2010; Pinjari, 2011; Bhat et al., 2013c), and (c) toward greater flexibility in the specification of the constraints faced by the consumer (Castro et al., 2012).

1.1 Gaps in Research

Despite the methodological advances and many empirical applications, one particular issue related to the budget constraint has yet to be resolved. Specifically, almost all KT model formulations in the literature, including the MDCEV model, assume that the available budget for total expenditure, i.e. y in Equation (1), is fixed for each individual (or for each choice occasion, if repeated choice data is available). Given the fixed budget, any changes in the decision-maker characteristics, choice alternative attributes, or the choice environment can only lead to a reallocation of the budget among different choice alternatives. The formulation itself does not allow either an increase or a decrease in the total available budget. Consider, for example, the context of households' vehicle holdings and utilization. In most applications of the KT approach for this context (Bhat et al., 2009, Ahn et al., 2008), a total annual mileage budget is assumed to be available for each household. This mileage budget is obtained exogenously for use in the KT model, which simply allocates the given total mileage among different vehicle types. Therefore, any changes in household characteristics, vehicle attributes (e.g., prices and fuel economy) and gasoline prices can only lead to a reallocation of the given mileage budget among the different vehicle types without allowance for either an increase or a decrease in the total mileage. Similarly, in the context of individuals' out-of-home activity participation and time-use, most applications of the KT approach consider an exogenously available total time

budget that is allocated among different activity type alternatives. The KT model itself does not allow either an increase or decrease in the total time expended in the activities of interest.

It is worth noting that the fixed budget assumption is not a theoretical/conceptual flaw of the consumer's utility maximization formulation per se. Classical microeconomics typically considered the consumption of broad consumption categories such as food, housing, and clothing. In such situations, all consumption categories potentially can be considered in the model while considering natural constraints such as total income for the budget. Similarly, several time-use analysis applications can use natural constraints individuals face as their time budgets (e.g., 24 hours in a day). However, many choice situations of interest involve the analysis of a specific broad category of consumption, with elemental consumption alternatives within that broad category, as opposed to all possible consumption categories that can possibly exhaust naturally available time and/or money budgets. For example, in a marketing context involving consumer purchases of a food product (say, yogurt), one can observe the different brands chosen by a consumer along with the consumption amount of (and expenditure on) each brand, but cannot observe the maximum amount of expenditure the consumer is willing to allocate to the product. It is unreasonable to assume that the consumer would consider his/her entire income as the budget for the choice occasion.

The above issue has been addressed in two different ways in the literature, as discussed briefly here (see Chintagunta and Nair, 2010; and von Haefen, 2010). The *first option* is to consider a two-stage budgeting process by invoking the assumptions of separability of preferences across a limited number of broad consumption categories and homothetic preferences within each broad category. The first stage involves allocation between the broad consumption categories while the second stage involves allocation among the elemental alternatives within the broad category of interest. The elemental alternatives in the broad consumption category of interest are called *inside goods*. The *second option* is to consider a Hicksian composite commodity (or multiple Hicksian commodities, one for each broad consumption category) that bundles all consumption alternatives that are not of interest to the analyst into a single *outside good* (or multiple outside goods, one for each broad consumption category). The assumption made here is Hicksian separability, where the prices of all elementary alternatives within the outside good vary proportionally and do not influence the choice and

expenditure allocation among the *inside goods* (see Deaton and Muellbauer, 1980). The analyst then models the expenditure allocation among all *inside goods* along with the *outside good*.

Many empirical studies use variants of the above two approaches either informally or formally with well-articulated assumptions. For instance, one can informally mimic the two-stage budgeting process by modeling the total expenditure on a specific set of choice alternatives of interest to the analyst in the first stage. The natural instinct may be to use linear (or log-linear) regression to model the total expenditure in the first stage. Subsequently, the second stage allocates the total expenditure among the different choice alternatives of interest. This approach is straightforward and also allows the total expenditure (in the first-stage regression) to depend on the characteristics of the choice-maker and the choice environment. The problem, however, is that the first-stage regression cannot incorporate the characteristics of choice alternatives in a straight forward fashion. Therefore, changes in the attributes of choice alternatives, such as price change of a single alternative, will only lead to reallocation of the total expenditure among choice alternatives without allowing for the possibility that the overall expenditure itself could increase or decrease. This is considered as a drawback in using the MDCEV approach for modeling vehicle holdings and usage (Fang, 2008) and for many other applications. Besides, from an intuitive standpoint, the observed expenditures may not necessarily represent the budget for consumption. It is more likely that a greater amount of underlying budget governs the expenditure patterns, which the consumers may or may not expend completely.

1.2 Current Research

This paper proposes the use of a stochastic frontier approach to estimate budgets for KT demand systems. Stochastic frontier models have been widely used in firm-production economics (Aigner et al, 1977; Kumbhakar and Lovell, 2000) for identifying the maximum possible production capacity (i.e., production frontier) as a function of various inputs. While the actual production levels and the inputs to the production can be observed, a latent production frontier is assumed to exist. Such a production frontier is the maximum possible production that can be achieved given the inputs.

In travel behavior research, the stochastic frontier approach has been used to analyze: (1) the time-space prism constraints that people face (Kitamura et al., 2000), and (2) the maximum amount of time that people are willing to allocate to travel in a day (Banerjee et al., 2007). In the former case, while the departure times and arrival times at fixed activities (such as work) are

observed in the survey data, the latest possible arrival time or the earliest possible departure time are unobserved and therefore modeled as stochastic frontiers. In the latter case, while the daily total travel time can be measured, an unobserved Travel Time Frontier (TTF) is assumed to exist that represents the maximum possible travel time an individual is willing to undertake in a day.

Analogous to the above examples, in many consumer choice situations, especially in time-use situations, one can conceive of latent time and/or money frontiers that govern choice making. Such frontiers can be viewed as the limit, or maximum amount of expenditure the individuals are willing to incur, or the expenditure budget available for consumption. We invoke this notion to use stochastic frontier models for estimating the budgets for consumption. Following the two-stage budgeting approach discussed earlier, the estimated budgets can be used for subsequent analysis of choices and allocations to different choice alternatives of interest. The same assumptions discussed earlier, such as weak separability of preferences, are needed here. However, an advantage of using the stochastic frontier approach over the traditional regression models (to estimate budgets) is that the frontier, by definition, is greater than the observed total expenditure. Therefore, the budget estimated using the stochastic frontier approach provides a “buffer” for the actual total expenditure to increase or decrease. This can be easily accommodated in the second stage consumption analysis (using KT models) by designating an *outside good* that represents the difference between the frontier and the actual expenditure on all the *inside goods* (i.e., choice alternatives of interest to the analyst). Given the frontier as the budget, if the attributes of the choice alternatives change, the second stage consumption analysis allows for the total expenditure on the inside alternatives to change (either increase or decrease). Specifically, within the limit set by the frontier, the outside good can either supply the additional resources (time/money) needed for inside goods or store the unspent resources. The theoretical basis of the notion of stochastic frontiers combined with the advantage just discussed makes the approach attractive for estimating the latent budgets for KT demand analysis.

As a proof of concept, we apply the proposed approach to analyze the daily out-of-home activity participation and time-use patterns in a survey sample of non-working adults in Florida. Specifically, we use the notion of an out-of-home activity time frontier (OH-ATF) that represents the maximum amount of time that an individual is willing to allocate to out-of-home (OH) activities in a day. First, a stochastic frontier regression is performed on the observed total out-of-home activity time *expenditure* to estimate the unobserved out-of-home activity time *frontier*

(OH-ATF). The estimated *frontier* is viewed as a subjective limit or maximum possible time individuals can allocate to OH activities and used to inform time budgets for a subsequent MDCEV model of activity time-use. Policy simulations are conducted to demonstrate the value of the proposed method in allowing the total out-of-home activity time *expenditure* to either expand or shrink within the limit of the *frontier* implied by the stochastic frontier model.

The efficacy of the proposed approach is compared with several other approaches to estimate budgets for the MDCEV model. Altogether, the following approaches are tested:

1. The stochastic frontier regression model for the total OH activity time frontier (OH-ATF),
2. A log-linear regression model to predict the total OH-activity time expenditure (OH-ATE),
3. Various assumptions on the time budget, without necessarily estimating it as a function of individuals' demographic characteristics. These include:
 - (3a) An arbitrarily assumed time budget of 875 minutes for every individual, which is equal to the total maximum observed OH-ATE in the sample plus 1 minute, and
 - (3b) An arbitrarily assumed time budget of 918 minutes for every individual, which is equal to 24 hrs minus an average of 8.7 hours of sleep time for non-workers (obtained from the 2009 American Time-use Survey),
 - (3c) An arbitrarily assumed time budget of 1000 minutes for every individual,
 - (3d) 24 hrs (1440 minutes) as the total time budget for every individual in the sample, and
 - (3e) 24 hrs minus observed in-home activity duration.

In the above approaches, the budget estimated using the log-linear regression approach is an estimate of the OH-activity time expenditure (OH-ATE), all of which is utilized for out-of-home activities. This is unlike the OH-ATF estimated using stochastic frontier regression, where the OH-ATF is by design greater than OH-ATE and therefore allows the specification of an *outside good* representing a portion of the frontier not spent in OH activities. As indicated earlier, the *outside good* allows for the total OH-ATE to increase or decrease due to changes in alternative-specific attributes. The other approaches listed above (3a to 3e) specify an arbitrary budget amount greater than the observed OH-ATEs.¹ Therefore, similar to the stochastic frontier approach, the analyst can specify an *outside good* in the time-use model to represent the

¹ Among the approaches 3a through 3e, all approaches except 3e assume an equal amount of budget across all individuals, while 3e allows the budget to be different across individuals depending on the differences in their in-home activities. While the approach 3e (i.e., utilizing 24 hrs minus in-home duration as the budget) does allow for different budgets across different individuals, it does not recognize the variation as a result of systematic demographic heterogeneity.

difference between the arbitrary budget and the total OH-ATE. The *outside good*, in turn, allows for the total OH-ATE to increase or decrease due to changes in alternative-specific attributes.

To compare the above-described approaches, seven different MDCEV models are estimated utilizing the time budgets estimated (or assumed) using the different approaches listed above – one MDCEV model for each approach. Subsequently, the time-use predictions from all the different MDCEV models are compared. The comparison is conducted both in terms of prediction accuracy against observed time-use patterns and the reasonableness of predicted changes in time-use patterns due to changes in alternative-specific variables.

Before moving forward with the analysis, it is worth noting a specific difference between the log-linear regression and stochastic frontier regression approaches to estimating budgets for the MDCEV model. Both the approaches allow for changes in time budgets due to changes in decision-maker characteristics and choice-environment attributes; i.e., log-linear regression allows changes in OH-ATE and stochastic frontier regression allows changes in OH-ATF. However, the stochastic frontier approach offers more flexibility when changes in alternative-specific attributes are considered. Such attributes could be attributes of the choice alternatives (e.g., prices per unit consumption) or choice environment attributes that influence the consumption of specific choice alternatives (e.g., accessibility to recreational land that might enter the MDCEV utility function for recreational activities). It is difficult, if not impossible, to include such attributes in the budget equations directly; i.e., in the log-linear or stochastic frontier regression equations. As a result, the time budgets (i.e., the OH-ATEs) estimated using log-linear regression remain the same between the base-case and the policy-case. The implication is that changes in alternative-specific attributes lead to a mere reallocation of the budget between different choice alternatives in the MDCEV model without allowing for the budget to increase or decrease. While the time budgets (i.e., the OH-ATFs) estimated using the stochastic frontier regression also do not change due to changes in alternative-specific attributes, as discussed earlier the ability to designate an *outside good* offers the flexibility for the total time expenditure on the *inside goods* (i.e., OH-ATE) to change.

The rest of the paper is organized as follows. Section 2 provides an overview of the stochastic frontier modeling methodology and the MDCEV model, in the current empirical context of OH activity time-use. Section 3 describes the Florida sample of the National

Household Travel Survey (NHTS) data used for the empirical analysis. Section 4 presents the empirical results and Section 5 concludes the paper.

2 METHODOLOGY

2.1 Stochastic Frontier Model for Out-of-home Activity Time Frontier

In the stochastic frontier approach used in this paper, the out-of-home activity time budget available to (or perceived by) an individual is assumed to be latent, and therefore called out-of-home activity time frontier (OH-ATF). While survey data provide measurements of actual out-of-home activity time expenditures (OH-ATE), they do not provide information about the upper bound of time people are willing to spend on activities out of home. The stochastic frontier modeling methodology is employed to model such an unobserved limit people perceive.

Following Banerjee et al. (2007), consider the notation below:

T_i = the observed total daily OH-ATE for person i ,

τ_i = the unobserved OH-AFT for person i ,

v_i = a normally distributed random component specific to person i ,

u_i = a non-negative random component assumed to follow a half-normal distribution,

X_i = a vector of observable individual characteristics,

β = a vector of coefficients of X_i ,

$\varepsilon_i = (v_i - u_i)$.

Let τ_i be a log-normally distributed unobserved OH-ATF of an individual i , while T_i is a log-normally distributed observed OH-ATE of the individual. Both these variables are assumed to be log-normally distributed to recognize the positive skew in the distribution of observed OH-ATE and to ensure positive predictions. τ_i of an individual is assumed to be a function of his/her demographic, attitudinal, and built environment characteristics, as:

$$\ln(\tau_i) = \beta' X_i + v_i \quad (2)$$

The unobserved OH-ATF can be related to the observed OH activity time expenditure T_i as:

$$\ln(T_i) = \ln(\tau_i) - u_i \quad (3)$$

Note that since u_i is non-negative, the observed OH-ATE is by design less than the OH-ATF.

Combining Equations (2) and (3) results in the following regression Equation:

$$\ln(T_i) = \boldsymbol{\beta}'\mathbf{X}_i + v_i - u_i = \boldsymbol{\beta}'\mathbf{X}_i + \varepsilon_i \quad (4)$$

In the above equation, the expression $\boldsymbol{\beta}'\mathbf{X}_i + v_i$ may be considered as representative of the location of the unobserved frontier for $\ln(T_i)$ with a random component v_i . Consistent with the formulation of the stochastic frontier model (Aigner et al, 1977), a half-normal distribution (with variance σ_u^2) is assumed for u_i and a normal distribution (with mean 0 and variance σ_v^2) is assumed for v_i . These two error components are assumed to be independent of one another to derive the probability density function of $\varepsilon_i (= v_i - u_i)$ as:

$$h(\varepsilon_i) = \frac{2}{\sqrt{2\pi\sigma}} \{1 - \Phi(\varepsilon_i \lambda / \sigma)\} \exp\left[-\frac{\varepsilon_i^2}{2\sigma^2}\right]; -\infty < \varepsilon_i < \infty \quad (5)$$

where, $\sigma^2 = \text{var}(v_i + u_i) = \sigma_v^2 + \sigma_u^2$, and $\lambda = \sigma_u / \sigma_v$. The ratio, λ , is an indicator of the relative variability of the sources of error in the model, namely v_i , which represents the variability among persons, and u_i , which represents the portion of the OH activity time frontier that remains unexpended (Aigner et al, 1977). The log likelihood function for the sample of observations is given by:

$$LL = \sum_{i=1}^n \ln[h(\varepsilon_i)] \quad (6)$$

Maximum likelihood estimation of the above function yields consistent estimates of the unknown parameters, $\boldsymbol{\beta}$, σ_u and σ_v .

From Equation (2), one can write OH-ATF as as: $\tau_i = \exp(\boldsymbol{\beta}'\mathbf{X}_i + v_i)$. Using this expression, once can compute the expected value of OH-ATF for individual i as:

$$E[\tau_i] = E\left[\exp(\boldsymbol{\beta}'\mathbf{X}_i + v_i)\right] = \exp\left(\boldsymbol{\beta}'\mathbf{X}_i + \frac{\sigma_v^2}{2}\right) \quad (7)$$

The expected OH-ATF may be used as the time budget in the second-stage analysis of activity participation and time-use.

2.2 MDCEV Model Structure for Out-of-Home Time-Use Analysis

The time-use model estimated in this study is based on Bhat's (2008) linear expenditure system (LES) utility form for the MDCEV model:

$$U_i(\mathbf{t}) = \sum_{k=1}^K \gamma_{ik} \psi_{ik} \ln \left(\frac{t_{ik}}{\gamma_{ik}} + 1 \right) \quad (8)$$

In the above function, $U_i(\mathbf{t})$ is the total utility derived by an individual i from his/her daily out-of-home activity participation and time-use. Individuals are assumed to choose their time-use patterns (i.e., which activities to participate in and how much time to allocate) to maximize $U(\mathbf{t})$ subject to a linear budget constraint on the available time for OH activity participation. The specification of this constraint depends on the approach used for the total available time budget. As discussed earlier, we tested three different approaches, as discussed next.

The first approach is the stochastic frontier approach, where the OH activity time *frontier* (τ_i) is used as the budget; i.e., the linear constraint then becomes $\sum_{k=1}^K t_{ik} = \tau_i$. In this paper, we use the expected value of OH-ATF as an estimate for τ_i , resulting in $\sum_{k=1}^K t_{ik} = E[\tau_i]$ as the actual budget constraint used in the time-use model. The second approach is to simply use the total activity time expenditure (T_i), which is observed in the data for model estimation purposes and can be estimated via a log-linear regression model for prediction purposes. In this case, the budget constraint would be $\sum_{k=1}^K t_{ik} = T_i$, where T_i is the total OH ATE. The third approach is to specify an arbitrarily assumed budget amount (greater than the observed OH-ATEs in the sample) on the right side of the budget constraint (i.e., approaches 3a to 3e discussed earlier).

In the above formulation, when the stochastic frontier approach is used to determine the budget, the first choice alternative ($k = 1$) in the utility function is designated as the *outside good* that represents the difference between the OH activity time frontier and the observed activity time expenditure (i.e., $t_1 = \tau_i - T_i$), while the other alternatives ($k = 2, 3, \dots, K$) are the *inside goods* representing different OH activities. Similarly, when an arbitrarily assumed budget (greater than the observed OH-ATEs) is used, the *outside good* represents the difference between the budget and the OH-ATE. On the other hand, when the OH-ATE (T_i) is itself used as the budget, there is no *outside good* in the formulation.

In the utility function, ψ_{ik} , labelled the baseline marginal utility of individual i for alternative k , is the marginal utility of time allocation to activity k at the point of zero time allocation. Between two choice alternatives, the alternative with greater baseline marginal utility

is more likely to be chosen. In addition, ψ_{ik} influences the amount of time allocated to alternative k , since a greater ψ_{ik} value implies a greater marginal utility of time allocation. γ_{ik} allows corner solutions (i.e., the possibility of not choosing an alternative) and differential satiation effects (diminishing marginal utility with increasing consumption) for different activity types. Specifically, when all else is same, an alternative with a greater value of γ_{ik} will have a slower rate of satiation and therefore a greater amount of time allocation.

The influence of observed and unobserved individual characteristics and built environment measures are accommodated as $\psi_1 = \exp(\xi_1)$, $\psi_k = \exp(\boldsymbol{\theta}'\mathbf{z}_k + \xi_k)$, and $\gamma_k = \exp(\boldsymbol{\delta}'\mathbf{w}_k)$; where, \mathbf{z}_k and \mathbf{w}_k are vectors of observed socio-demographic and activity-travel environment measures influencing the choice of and time allocation to activity k , $\boldsymbol{\theta}$ and $\boldsymbol{\delta}$ are corresponding parameter vectors, and ξ_k ($k=1,2,\dots,K$) is the random error term in the sub-utility of activity type k . Assuming that the random error terms ε_k ($k=1,2,\dots,K$) follow the independent and identically distributed (iid) standard Gumbel distribution leads to a simple probability expression (see Bhat, 2005) that can be used in the familiar maximum likelihood routine to estimate the unknown parameters in $\boldsymbol{\theta}$ and $\boldsymbol{\delta}$.

3 DATA

The time-use data used in this paper comes from the Florida add-on of the US National Household Travel Survey (NHTS). The empirical focus is on adult non-workers' out-of-home (OH) activity time-use on weekdays. The travel information collected in the survey was used to determine daily time allocation to eight OH activities – shopping, personal business, social/recreation, active recreation, medical visits, eat out, pickup/drop-off, and other activities.

Table 1 provides descriptive information on the estimation sample used in this analysis. The sample comprises 6,218 individuals who participated in at least one out-of-home activity on the survey-day. Only the interesting characteristics of the sample are discussed here for brevity. A large portion of the sample comprises elderly; partly due to a large share of elderly in Florida's population and also due to a skew in the response rates of different age groups to the survey. The dominant share of elderly in the sample is perhaps a reason for a greater share of females (than males), a higher than typical proportion of smaller size households, larger share of households

without children and those with no workers, and predominantly urban residential locations. A large share of the sample is Caucasian, able to drive, and owns at least one vehicle in the household. Several other demographic variables reported in the table are relevant to the models estimated in this paper.

We compared the demographic characteristics presented in the table (specifically, age, gender, and race) with the state level non-worker population demographics obtained from the American Community Survey (ACS) (ACS estimates are not shown in the table). The comparison revealed that the current data sample has an overrepresentation of elderly individuals (perhaps due to differences in the response rates of individuals from different age groups to the survey). In addition, the proportion of Caucasians is higher in the sample than that from the ACS data. The gender distribution in the data was similar to that from the ACS data. While the data sample may not be fully representative of the non-worker population in the state, the empirical analysis presented in this paper can still serve as a proof of concept; of course, the empirical results must be used in caution in the context of policy discussions.

In addition to the demographic variables, a variety of different residential land-use characteristics were considered for explaining activity participation and time allocation decisions. These include housing and employment density measures, dummy variables for urban and rural areas, accessibility measures, activity opportunity variables (such as employment of different types within buffers of 0.25 miles, 0.5 miles, and 1 miles around the household), transportation network variables (such as roadway miles per square mile, number of intersections per square mile, and number of cul-de-sacs per square mile around the household). Of all these variables, the descriptive statistics of those that appear in the empirical model specifications are reported in Table 1. As can be observed, only the accessibility (to recreational land) variable shows a considerably higher variation (i.e., standard deviation) than its sample mean. The other two variables – employment within 1 mile buffer of household and number of cul-de-sacs within a quarter mile of the household – have a slightly higher standard deviation than their respective sample mean values. The empirical distributions (histograms) of these variables in the sample (not reported in the paper) look akin to an exponential distribution, beginning with large proportions of small values and ending with small proportions of large values. A higher proportion of the data concentrated around smaller values is a reason for a small variation of

land-use characteristics in the data. Such small variations in the data will have a bearing on estimating the effects of land-use characteristics on activity participation and time allocation.

The last part of the table presents the OH activity participation and time-use statistics observed in the sample. On average, individuals in the sample spent around two-and-half hours on OH activities. Majority of them participated in shopping activities, followed by personal business, social/recreation, eat out, medical, active recreation, pickup/drop-off, and other activities. Note that the percentages of participation in different activities add up to more than 100, because a majority of individuals participate in multiple activities. On average, individuals in the sample participated in 2.6 OH activities; 32% participated in two activities and 36% participated in at least 3 activities. This calls for the use of the multiple-discrete choice modeling approach for modeling time-use. In terms of time allocation, those who participate in social recreation do so for an average of 2 hours. The average time allocation to shopping, personal business, active recreation, eat out, or medical activities ranges from 45 minutes to an hour, while that for pickup/drop-off and other activities is around 15 minutes.

While not reported in the tables, some useful patterns observed in the data and relevant to the modeling results presented later are: (a) greater proportion of females participate in shopping and social/recreation activities and for larger durations, (b) older people participate more in medical activities while younger people participate more in social/recreational activities, (c) those with a driver's license are likely to do more out of home activities, especially pickup/drop-off, (d) those with children undertake more pickup/drop-off activities, and (e) higher income individuals participate more in social and active recreation and eat out activities. In summary, the sample shows reasonable time allocation patterns that are typical of the non-working population in Florida.

4 EMPIRICAL RESULTS

4.1 Stochastic Frontier Model of Out-of-Home Activity Time Frontier (OH-ATF)

Table 2 presents the results of the stochastic frontier model for OH-ATFs. Interestingly, female non-workers are found to have larger OH ATFs than male non-workers in Florida. Upon closer examination, this result can be traced to larger participation of females in shopping and social/recreation activities that tend to be of larger duration. As expected, the frontier is larger for people of younger age groups and for those who have driver licenses. Blacks seem to have larger

frontiers than Whites and others; see Banerjee et al. (2007) for a similar finding. Internet use is positively associated with OH-ATF. People from single person households, high income households, and zero-worker households tend to have larger OH-ATFs; presumably because of the greater need for social interaction for single-person households, greater amount of money among higher income households to buy home maintenance services and free-up time for OH activity (as well as greater affordability to consume OH activities), and lower time-constraints of zero-worker households. People living in urban locations have larger OH-ATFs than those in rural locations, perhaps due to a greater presence of OH activity opportunities in urban locations. Mondays are associated with smaller perceived frontiers for OH non-worker activity, possibly due to pronounced OH activity pursued over the weekend just before Monday and also due to the effect of Monday being the first work day of the week. Several other demographic variables were explored but turned non-influential in the final model. These include education status, vehicle ownership, presence of children, and own/rent house. This may be because the income effects in the model act as surrogate for many of these variables. Several land-use and built-environment variables, except an indicator for urban/rural location of the household residence, also turned out statistically non-influential in the model.

The stochastic frontier model estimates can be used to estimate the expected OH-ATF (see Eq. 7) for each individual in the survey sample to generate a distribution of expected ATFs. The average value of the expected ATF in the estimation sample is 400 minutes (6.5 hours), whereas the average total OH time expenditure is 152 minutes (about 2.5 hours), suggesting that people are utilizing close to 40% of their perceived time budgets for OH activity. Of course, the percentage utilization varies significantly with greater utilization for those with larger observed OH activity expenditures and smaller utilization for those with smaller observed expenditures.

The goodness of fit of the stochastic frontier model may be evaluated using a log-likelihood ratio test to assess how well the demographic and land-use explanatory variables in the model explain the variation in OH-ATFs over a constant only model that doesn't include any of these explanatory variables. The log-likelihood value for the constant only model is -8739.99 and that of the final empirical specification with 12 demographic and land-use explanatory variables is -8675.99. The log-likelihood ratio between the two models is 128, which is far greater than the critical chi-square value of 26.217 for 12 degrees of freedom at a 99%

confidence level. This suggests the importance of the demographic and land-use variables included in the model to explain variations in OH-ATFs.

Note that the goodness of fit of the empirical model might further be improved using additional variables describing transportation level of service (i.e., how easy is it to travel for OH activities) and individuals' attitudes towards OH activities. Such variables were not available in the empirical data from NHTS. When we explored the influence of other variables (that were available in the data) such as educational attainment of the individuals and residential land-use variables such as accessibility to recreational sites and employment within a mile buffer of the household, we did not find a statistically significant influence of those variables. Further work is necessary to identify the influence of additional demographic, land-use, and transportation level of service variables on OH-ATFs.

It is worth noting here that the log-linear regression model (on the observed OH-ATEs) provided similar substantive interpretations of the impacts of individual and household demographic variables on OH-ATEs, albeit with different parameter estimates. Therefore those results are not discussed exclusively here.

4.2 Out-of-home Activity Time-use Model Results

We estimated seven different MDCEV models of time-use with different assumptions discussed earlier on time budgets. Overall, the parameters estimates from all the models were found to be intuitive and consistent in interpretation with each other and previous studies. For brevity, this section presents (in Table 3) and discusses only the results of the model in which the expected OH-ATFs (estimated using the stochastic frontier approach) were used as the time budgets. Note that the statistical significance of parameter estimates was determined at 80% confidence level, because of the small data sample.

The baseline utility parameters suggest that females are more likely (than males) to participate in shopping and pickup/drop-off activities but less likely to participate in active recreation, albeit the influence is not statistically significant even at 80% confidence level.² With increasing age, social/recreational activities and pickup/drop-off activities reduce, while medical visits increase. As expected, licensed drivers are more likely to participate in all OH activities

² The female variable was retained in the baseline utility (with a p-value slightly below 80% confidence level) because, as discussed later, this variable appears in the satiation function with a statistically significant coefficient. Without including the female variable in the baseline utility function, the influence of the same variable would be overestimated in the satiation function.

(i.e., they are likely to use a large proportion of their frontiers) and even more so for pickup/drop-off activities. Reflecting cultural differences, Whites are more likely to eat out than those from other races while those born in the US are more likely to eat, socialize and recreate out-of-home than immigrants. Individuals with a higher education attainment are more likely to undertake personal business (e.g., buy professional services) and active recreation. Those from households with children and households with more workers show lower participation in shopping and personal business but do more pickup/drop-off activities. Income shows a positive association with social/recreational activities, active recreation, and eating out; however, the income differences are not significant even at 80% confidence level. Several land-use variables were attempted to be included in the model but only a few turned out marginally significant, perhaps because of a small variation of land-use characteristics across the sample. Among these, accessibility to recreational land seems to encourage social recreation as well as active recreation; employment density (measured by # jobs within a mile of the household) and # cul-de-sacs within a quarter mile buffer (a surrogate for smaller amount of through traffic) are positively associated with active recreation. It remains to be seen, as explored later using policy simulations, if these variables have a practically significant influence on time-use. Finally, Monday is associated with smaller rates of social recreation and eat-out activities while Fridays attract higher rates of social recreation, albeit the influence of Fridays is not statistically significant at 80% confidence level. Note that the baseline utility function for unspent time alternative (i.e., the *outside good*) does not have any observed explanatory variables in it, as the alternative was chosen as the base alternative for parameter identification in the utility functions of OH alternatives.

The satiation function parameters influence the continuous choice component; i.e., the amount of time allocation to each activity. The relative magnitudes of the satiation function constants are largely consistent with that of the observed durations for different activities. For example, social recreational activities have a high satiation constant suggesting they are more likely to be pursued for longer durations. The unspent time alternative has the largest satiation constant reflecting that large proportions of the perceived OH-activity time frontiers in the sample are unspent. Females tend to allocate more time to shopping and social recreation but less time to active recreation, if they participate in these activities. People from middle age group tend to spend less time in social/recreation, while educational attainment is associated with larger

time in active recreation. Mondays tend to have smaller time allocations for eating out, while Fridays are associated with larger time allocations to social/recreation and eating out. Finally, accessibility to recreational land has a positive, but statistically insignificant (at 80% confidence level) influence on the time allocation to social/recreation and active recreation.

The log-likelihood value for the MDCEV model with only constants (*i.e.*, with no observed socio-demographic and land-use variables in the utility specification) is -105505. The log-likelihood value at convergence for the final model specification presented here with an additional 49 estimated parameters is -105087. The log-likelihood ratio index between these two values is 835.38, which is larger than the critical chi-square value with 49 degrees of freedom at any reasonable level of significance. This suggests the importance of the demographic and land-use variables included in the model to explain the observed variation in the time-use choices.

4.3 Predictive Accuracy Assessments on the Estimation Sample

This section presents a comparison of in-sample predictive accuracy assessments for the different MDCEV models estimated in this study based on different assumptions for OH activity time budgets. All predictions with the MDCEV model were undertaken using the procedures proposed by Pinjari and Bhat (2011), using 100 sets of Halton draws to cover the error distributions for each individual in the data.

Table 4 presents the results. Specifically, the observed and predicted activity participation rates are presented in the top part of the table, while the observed and predicted activity durations are presented in the bottom part. The predicted participation rates for each activity were computed as the proportion of the instances the activity was predicted with a positive time allocation across all 100 sets of random draws for all individuals. The predicted average duration for an activity was computed as the average of the predicted duration across all random draws for all individuals with a positive time allocation. In the rows labeled “mean absolute error”, an overall measure of error in the aggregate predictions is reported. This measure is an average, across different activities, of the absolute difference between observed aggregate values and the corresponding aggregate predictions. Several interesting observations can be made from these results. First, the MDCEV models that use budgets from the stochastic frontier model or the log-linear regression model exhibit a greater aggregate-level predictive accuracy than other MDCEV models. This is presumably because the budgets used for both the models are heterogeneous across individuals (based on their demographic characteristics), whereas other approaches do not

systematically capture heterogeneity in the available time budgets across individuals. These results suggest the importance of capturing demographic heterogeneity in the available time budgets across different individuals for a better prediction of the daily activity participation and time-allocations by the MDCEV time-use model. Second, between the stochastic frontier and log-linear regression approaches, quality of the aggregate predictions is similar; albeit the predicted activity participation rates for the stochastic frontier approach are slightly better, while the predicted activity durations for the log-linear regression approach are slightly better. Third, the predictive accuracy does not seem to differ significantly by the amount of total budget assumed if a constant amount is used as the budget for every individual in the sample. Specifically, the predictions were very similar between the models that assumed an equal amount of budget across all individuals – 875 minutes, 918 minutes, 1000 minutes, or 24 hours – albeit there seems to be deterioration in the predictions as the assumed budget amount increases.

4.4 Predictive Accuracy Assessments on a Holdout Sample

The predictive accuracy assessments presented in the previous section were not on a holdout sample. In this section we present predictive accuracy assessment on a holdout sample. To do so, we split the entire data sample (of 6,218 individuals) into an estimation sample of 5,218 individuals and a validation sample (i.e., holdout sample) of 1,000 individuals. Both, first-stage, time budget models (stochastic frontier and log-linear regression models) and second-stage, time use models were estimated using the sample of 5,218 individuals. The parameter estimates obtained from the estimation sample of 5,218 individuals were used to predict the time allocations in the holdout sample of 1,000 individuals. To conserve space, these model estimation results (i.e., those from 5,218 individuals) are not reported in the paper, but available from the authors. The predictive assessment results on the hold-out sample are presented in Table 5, in a similar format as that in Table 4. Very similar to the results in Table 4, and as discussed in the previous section, the MDCEV model predictions using time budgets from log-linear regression and stochastic frontier regression approaches outperform those from other approaches. Between the log-linear regression and stochastic frontier regression approaches, the predicted activity participation rates for the stochastic frontier approach are slightly better, whereas the predicted activity durations for the log-linear regression approach are slightly better.

To further examine prediction accuracy in the context of the activity durations (i.e., the continuous choice component), Figure 1 presents the distributions of observed and predicted

distributions of activity durations for different MDCEV models in the form of box plots. To conserve space, the box plots are provided for six out of eight OH activities modeled in this paper. One can observe from this figure that the predictions from the log-linear regression and stochastic frontier regression approaches match better with the observed distributions than predictions from arbitrarily assumed budgets. Between log-linear regression and stochastic frontier regression approaches, the former approach appears to perform slightly better for most activities. The stochastic frontier regression shows a greater tendency to overestimate the activity durations. This is expected because the log-linear regression approach restricts the time budget to only the time allocated to OH activities of interest, whereas the stochastic frontier approach allows an *unspent* part in the time budget. Given a larger amount of time budget available from the stochastic frontier approach, the predicted time allocations to OH activities are likely to be overestimated. The important point to note, however, as demonstrated in the next section, is that the *unspent* alternative offers a way for the total OH activity time expenditure to expand or shrink due to changes in alternative specific variables.

4.5 Comparison of Policy Simulations

This section presents the predictions of a hypothetical policy scenario using the different MDCEV models estimated in this study based on different approaches for time budgets.³ The policy scenario considered in this exercise is doubling of accessibility to recreational land-use. To simulate the effects of this hypothetical policy, in the first step, time budgets were estimated for both the base-case and the policy-case (i.e., before-policy and after-policy, respectively).⁴ However, since the corresponding variable – accessibility to recreational land – does not appear in either the log-linear regression or the stochastic frontier regression equations, the estimated time budgets do not differ between the base-case and the policy-case. Similarly, the time-budget remains the same between the base-case and the policy-case when an arbitrarily assumed deterministic time-budget is used (i.e., approaches 3a to 3e in Section 1). In the second step, the time budgets from the first step were used as budgets for the corresponding MDCEV time-use models (along with the MDCEV parameter estimates) to simulate out-of-home time-use patterns

³ The policy simulations were conducted on a full estimation sample of 6,218 individuals using the parameter estimates obtained from this sample.

⁴ For the log-linear regression and stochastic frontier regression approaches, the time budgets were estimated by simply taking the expected value of the corresponding regression equations. For other approaches where deterministic amounts of time budgets were assumed for all individuals in the sample (i.e., approaches 3a to 3e in Section 1), those same assumptions were used for prediction as well.

in the base-case and policy-case. Subsequently, the policy effect was quantified as two different measures of differences in time-use patterns between the policy-case and base-case: (1) The percentage of individuals for whom the time allocation to different activities changed by more than a minute⁵, and (2) The average change in time allocation for whom the time allocation changed by more than a minute. Table 6 reports these measures for the different approaches/assumptions used in the study for estimating time budgets. Specifically, in each row (i.e., for each approach used to estimate time-budget) for each column (i.e., for an activity type), the % number represents the percentage of individuals for whom the time allocated to the corresponding activity changed by more than a minute. The number in the parenthesis adjacent to the % figure is the average change in time allocation (in minutes) for whom the time allocation to that activity changed by more than a minute. Several observations can be made from this table, as discussed next.

First, across all approaches for arriving at time budgets, consistent with the MDCEV model parameter estimates, increasing accessibility to recreational land-use has increased the time allocation to OH social and active recreational activities. For example, with the stochastic frontier approach for time budgets, doubling accessibility to recreational land lead to an increased time allocation (by more than a minute) for 3% individuals in social recreation activities and for 2.2% individuals in active recreation activities. Among these individuals, on average, the time spent in social recreation increased by 21 minutes and that in active recreation increased by 25 minutes, respectively.

Second, upon examining where the additional time for social and recreational activities comes from, the MDCEV model based on the log-linear regression approach for time budgets differs considerably from the other MDCEV models. Specifically, using estimated OH-ATEs from the log-linear regression as budgets leads to a simple reallocation of the time (i.e., the estimated OH-ATE) between different activity types. That is, all of the increase in time allocation to social and recreational activities must come from a decrease in the time allocation to other activities. This is a reason why the predicted increases in the social and recreational activity participation rates are the smallest (and for a smaller percentage of individuals) for the log-linear regression approach. On the other hand, the stochastic frontier approach provides a

⁵ We report only those for whom the time allocation changed by more than a minute (and the average change in time allocation only for those individuals) as opposed to all individuals for whom the time allocation changed. This helps in avoiding the consideration of instances when changes in time allocation are negligible (i.e., less than a minute).

“buffer” in the form of an unspent time alternative from where the additional time for social and active recreational pursuits can be drawn. Therefore, the increase in the time allocation to social and active recreational activities comes partly from a reduction in the “unspent time” and partly from other OH activities. This reflects an overall increase in the total OH activity expenditure (OH-ATE) than a mere reallocation of the base-case OH-ATE. Such an increase in the total OH-ATE can be measured by the decrease in the time allocated for the “unspent time” alternative; for example, an average of 21 minutes for the stochastic frontier approach. Intuitively speaking, it is reasonable to expect that an increase in accessibility to recreational land would lead to an increase in social and active recreation activity and there by an overall increase in OH activity time among non-workers, as opposed to a mere reallocation of time across different OH activities. This demonstrates the value of the stochastic frontier approach in allowing more reasonable effects of changes in alternative-specific explanatory variables in the MDCEV model.

Third, similar to the stochastic frontier approach, other approaches that assume an arbitrary budget greater than observed OH-ATEs also allow a “buffer” alternative. In fact, the policy forecasts from all these approaches are similar to (albeit slightly higher than) those from the stochastic frontier approach. But recall that their base-case predictions (against observed time-use patterns) were inferior compared to the stochastic frontier approach. Therefore, it might be better to use the stochastic frontier approach than making arbitrary assumptions on the time budgets.

5 SUMMARY AND CONCLUSIONS

This paper presents a stochastic frontier approach to estimate budgets for the multiple discrete-continuous extreme value (MDCEV) model. The approach is useful when the underlying time and/or money budgets driving a choice situation are unobserved, but only the expenditures on the choice alternatives of interest are observed. Most MDCEV applications hitherto used the observed total expenditure on the choice alternatives as the budget to model the pattern of expenditure allocation among different choice alternatives. This does not allow the possibility that changes in choice alternative attributes can lead to changes in the total expenditure, but only allows a reallocation of the observed total expenditure among the choice alternatives. The stochastic frontier approach resolves this issue by invoking the notion that consumers operate under latent budgets that can be conceived (and modeled) as the maximum possible expenditure

they are willing to incur. The estimated stochastic frontier, or the subjective limit, or the maximum amount of expenditure consumers are willing to allocate can be used as the budget in the MDCEV model. Since the frontier is by design larger than the observed total expenditure, the MDCEV model needs to include an *outside alternative* along with all the choice alternatives of interest to the analyst. The outside alternative represents the difference between the frontier (i.e., the budget) and the total expenditure on the choice alternatives of interest. The presence of this outside alternative allows for the total expenditure on the inside alternatives to increase or decrease as a result of changes in decision-maker characteristics, choice environment attributes, and, more importantly, the choice alternative attributes.

As a proof of concept, the proposed approach is applied to analyze the daily out-of-home activity participation and time-use patterns in a survey sample of non-working adults in Florida. Specifically, we use the notion of an out-of-home activity time *frontier* (OH-ATF) that represents the maximum amount of time that an individual is willing to allocate to out-of-home (OH) activities in a day. First, a stochastic frontier regression is performed on the observed total out-of-home activity time *expenditure* (OH-ATE) to estimate the unobserved out-of-home activity time *frontier* (OH-ATF). The estimated frontier is viewed as a subjective limit or maximum possible time individuals are willing to allocate to out-of-home activities and used to inform time budgets for a subsequent MDCEV model of activity time-use. The efficacy of the proposed approach is compared with the following other approaches to estimate budgets for the MDCEV model:

- (a) Using total OH-activity time expenditure (OH-ATE), estimated via log-linear regression, as the time budget, and
- (b) Various assumptions on the time budget, without necessarily estimating it as a function of individual's demographic and built environment characteristics.

The comparisons were based on predictive accuracy (on both the estimation sample and a holdout sample) and reasonableness in the results of hypothetical scenario simulations of changes in land-use. The overall findings from this empirical exercise are summarized below.

- Employing time budgets obtained from the stochastic frontier approach (to estimate OH-ATF) and the log-linear regression approach (to estimate the OH-ATE) provide better predictions of OH activity and time-use patterns from the subsequent MDCEV models, than employing arbitrarily assumed time budgets. This is presumably because the former

approaches allow for the time budgets to vary systematically based on individual's demographic characteristics, while the latter approaches assume an arbitrary budget that does not allow demographic variation in the budgets.

- Between the log-linear regression and stochastic frontier regression approaches, the predicted activity participation rates for the stochastic frontier approach were relatively better, while the predicted activity durations for the log-linear regression approach were relatively better. Using the latter approach resulted in a slightly greater tendency to overestimate activity durations.
- While both the stochastic frontier and the log-linear regression approaches provided similar prediction performance (at the aggregate level), the former approach allows for the total OH activity time expenditure to increase or decrease due to changes in alternative-specific variables. On the other hand, using time budgets from the log-linear regression approach lead to a mere reallocation of time between the different OH activities without increasing the total time allocated for OH activities. This is an important advantage of the stochastic frontier approach over the traditional log-linear regression approach to estimating activity time budgets.
- When arbitrarily assumed time budgets were considered, the predictive accuracy and policy simulation outcomes (in terms of the changes in OH time allocation patterns) did not differ significantly between the different assumptions as long as an equal time budget was assumed for all individuals.

Overall, the empirical results demonstrate the value of the proposed stochastic frontier approach to estimating unobserved budgets for the MDCEV models. While the current empirical application is in the context of time-use, the proposed approach is applicable to estimate budgets for many empirical applications involving MDC choice analysis, including household vehicle holding and usage, long-distance vacation time and money budgets, and market basket analysis. However, the current empirical analysis should be viewed as only a proof of concept. Additional empirical analyses with a variety of different empirical contexts and data sets will be beneficial (see, for example, Augustin et al., 2015 for an assessment in the context of household vehicle holding and usage). Finally, since many land-use variable effects on time-use were not significant in the current empirical analysis, it will be interesting to conduct empirical analyses in

different geographical contexts with a greater variation in land-use characteristics or with empirical data gathered from a variety of different urban development patterns.

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TABLE 1. Descriptive Statistics of the Estimation Sample

Person Characteristics		Household Characteristics							
Sample Size	6,218	Sample Size	4,766						
Age		Household Size							
18 - 24 years	1.40%	1 Person	24.60%						
25 - 64 years	33.80%	2 Person	55.80%						
65+ years	64.70%	3+ Person	19.60%						
Gender		Annual Income							
Male	42.80%	< \$ 25 K	29.00%						
Female	57.20%	\$ 25 K - \$50 K	33.20%						
		\$ 51 K - \$75 K	15.30%						
		> \$ 75K	22.60%						
Race		Number of Workers							
White	90.30%	0 Workers	69.50%						
African American	5.30%	1 Worker	26.50%						
Other	4.4%	2+ Workers	5.00%						
Education Level		Number of Drivers							
High School or less	40.80%	0 Drivers	2.90%						
Some College	28.40%	1 Driver	31.80%						
Bachelor/Higher	30.80%	2 Drivers	56.40%						
		3+ Drivers	8.90%						
Driver Status		Number of Children							
Driver	91.70%	0 Children	90.10%						
Not a Driver	8.30%	1 Child	5.00%						
		2+ Children	4.90%						
Internet Use		Residential Land-Use Variables							
Almost Everyday	46.30%	Accessibility to recreational land in 5mile buffer	Mean = 1.94 (St. dev = 6.88)						
Several Times in a week	10.30%	# Employments within 1 mile buffer of HH	Mean = 32.75 (St. dev = 47.38)						
Sometimes (once in a week or in a month)	6.40%	# Cul-de-sacs within 0.25 mile buffer of HH	Mean = 4.55 (St. dev = 5.08)						
Never	37.00%	Residential area type is urban (not rural)	78.90%						
Persons' Daily Out-of-Home Activity Participation and Time-Use Characteristics									
	Total observed OH Activity Duration	Shopping	Personal Business	Social/ Recreational	Active Recreation	Medical	Eat Out	Pick-Up/ Drop Off	Other
% Participation	100	63.3	39.1	37.6	26.3	29.9	32.5	20.1	7.7
Average daily activity duration in minutes (St. dev in parenthesis)	152.8 (120.9)	54.6 (50.6)	49.6 (57.3)	124.1 (102.6)	52.7 (81.8)	60.1 (70.0)	47.9 (42.3)	15.3 (23.9)	20.8 (43.2)

Notes: Reported activity durations are averages among those who participated in the activity. Numbers in the parentheses are standard deviations (for residential land-use and for activity duration variables).

TABLE 2. Parameter Estimates of Out-of-Home Activity Time Frontier (OH-ATF) Model

Variables	Coefficients (t-stats)
Constant	6.03(138.28)
Female	0.08(3.97)
Young age; 18-29 years (mid age is base)	0.11(1.89)
Old age; >75 years (mid age is base)	-0.08(-3.48)
Black (white and others are base)	0.09(2.12)
Licensed to drive	0.12(3.46)
Uses internet at least once a week (no use is base)	0.08(3.48)
Single person household	0.19(4.96)
Low income < 25K/annum (medium income is base)	-0.07(-2.92)
High income >75K/annum (medium income is base)	0.05(2.00)
Zero-worker household	0.07(2.73)
Urban residential location (rural is base)	0.04(1.87)
Monday (Tuesday - Friday is base)	-0.09(-3.74)
$\hat{\sigma}_u$	1.7164 (84.97)
$\hat{\sigma}_v$	0.2851 (23.37)
Log-likelihood at constants	-8739.99
Log-likelihood at convergence	-8675.99
No. of parameters estimated	15

Note: For all parameter estimated in this table, the t-statistic value corresponds to at least 90% confidence interval.

TABLE 3. Parameter Estimates of MDCEV Out-of-Home Activity Time-Use Model with Budgets from the Stochastic Frontier Approach

	Unspent Time	Shopping	Personal Business	Social Rec.	Active Rec.	Medical	Eat Out	Pickup /Drop	Other
Baseline Utility Variables									
Constants	-	-1.03(-14.67)	-1.87(-26.23)	-2.10(-21.00)	-2.57(-31.11)	-2.39(-26.65)	-2.91(-23.32)	-2.92(-17.16)	-3.74(-48.29)
Female (Male is base)	-	0.05(1.24)**	-	-	-0.11(-2.08)	-	-	0.09(1.59)*	-
Age <30 years (30-54 is base)	-	-	-	0.59(4.82)	-	-	-	-	-
Age 55-64 years	-	-	-	-	-	0.10(1.21)**	-	-0.30(-3.17)	-
Age 65-74 years	-	-	-	-	-	0.14(1.78)*	-	-0.43(-4.60)	-
Age >= 75 years	-	-	-	-0.07(-1.33)*	-	0.32(4.15)	-	-0.60(-6.17)	-
White (Non-white is base)	-	-	-	-	-	-	0.39(4.09)	-	-
Driver (Non-driver is base)	-	-	-	-	-	-	-	0.48(3.33)	-
Driver (All OH activities)	-	0.28(4.72)	0.28(4.72)	0.28(4.72)	0.28(4.72)	0.28(4.72)	0.28(4.72)	0.28(4.72)	0.28(4.72)
Some College (< college is base)	-	-	0.16(2.96)	-	-	-	-	-	-
Bachelors degree or more	-	-	0.25(4.74)	-	0.28(4.87)	-	-	-	-
Born in US (others is base)	-	-	-	0.11(1.63)*	-	-	0.30(3.86)	-	-
# Children aged 0-5 years	-	-0.12(-1.80)	-0.23(-2.68)	-	-	-	-	0.38(5.29)	-
# Children aged 6-15 years	-	-	-	-	-	-	-	0.46(9.01)	-
Total number of workers	-	-0.04(-1.25)**	-	-	-	-	-	0.16(3.20)	-
Income 25- 50 K	-	-	-	0.06(1.12)**	-	-	0.24(3.74)	-	-
Income 50-75 K	-	-	-	0.06(1.12)**	0.21(2.84)	-	0.28(3.65)	-	-
Income >75 K	-	-	-	0.06(1.12)**	0.41(6.47)	-	0.45(6.51)	-	-
Accessibility to recreational land	-	-	-	0.0059(1.84)	0.0052(1.45)*	-	-	-	-
# Employments (1 mile buffer)	-	-	-	-	0.0009(1.96)	-	-	-	-
# Cul-de-sacs (0.25 mile buffer)	-	-	-	-	0.007(1.29)*	-	-	-	-
Monday (Tue.-Thurs.is base)	-	-	-	-0.14(-2.35)	-	-	-0.21(-3.22)	-	-
Friday (Tue.-Thurs.is base)	-	-	-	0.06(1.11)**	-	-	-	-	-
Satiation Function Variables									
Constants	4.66(109.28)	2.83(63.91)	3.01(86.02)	4.42(88.96)	1.60(15.88)	3.27(76.43)	3.14(63.06)	1.45(30.21)	2.22(30.58)
Female (Male is base)	-	0.24(4.14)	-	0.12(2.02)	-0.13(-1.33)*	-	-	-	-
30-54 years(<30 & >55 years-base)	-	-	-	-0.27(-2.52)	-	-	-	-	-
Some College (< college is base)	-	-	-	-	0.45(3.66)	-	-	-	-
Bachelors degree or more	-	-	-	-	0.76(6.46)	-	-	-	-
Monday (Tue.-Thurs. is base)	-	-	-	-	-	-	-0.19(-1.76)	-	-
Friday	-	-	-	0.12(1.22)*	-	-	0.25(2.57)	-	-
Accessibility to recreational land	-	-	-	0.005(0.91)**	0.023(3.39)	-	-	-	-
Model goodness-of-fit	Log-likelihood at constants = -105505; Log-likelihood for the final specification = -105087.31; Total no of parameters estimated = 66 (17 of these parameters are constants).								

Notes: t-statistics are reported in parentheses. **t-statistic value is for less than 80% confidence interval. * t-statistic value is for 80% to 90% confidence interval. For all other parameters, the t-statistic value is for at least 90% confidence interval.

TABLE 4. In-Sample Predictive Performance of MDCEV Time-use Models with Different Approaches for Time Budgets

	Observed and predicted activity participation rates							
	Observed	Log-linear Regression	Stochastic Frontier	Budget = 875 min.	Budget = 918 min.	Budget = 1000 min.	Budget = 1440 min. (24 hrs.)	Budget = 24hrs-in home duration
Shopping	63.3%	67.1%	58.0%	56.0%	55.9%	55.7%	55.4%	53.7%
Personal Business	39.1%	45.9%	37.3%	35.9%	35.9%	35.9%	35.9%	34.2%
Social Recreation	37.6%	43.4%	34.7%	33.6%	33.5%	33.5%	33.4%	31.8%
Active Recreation	26.3%	27.8%	23.3%	22.5%	22.5%	22.5%	22.6%	21.1%
Medical	29.9%	32.1%	26.6%	25.5%	25.5%	25.5%	25.6%	24.0%
Eat Out	32.5%	35.9%	29.5%	28.4%	28.4%	28.4%	28.5%	27.0%
Pickup /Drop-off	20.1%	22.3%	18.4%	17.7%	17.7%	17.7%	17.9%	16.7%
Other Activities	7.7%	8.1%	6.8%	6.5%	6.5%	6.5%	6.6%	6.0%
Mean Absolute Error	-	3.28	2.74	3.80	3.83	3.86	3.83	5.25
	Observed and predicted average activity durations (minutes) for those who participated in the activity							
Shopping	54.6	65.0	78.7	92.7	93.9	96.4	107.7	86.7
Personal Business	49.6	50.8	64.2	76.0	76.9	78.8	87.5	71.7
Social Recreation	124.1	87.8	131.8	160.7	162.9	167.5	189.2	150.0
Active Recreation	52.7	26.4	30.5	35.0	35.3	35.9	38.8	33.0
Medical	60.1	53.2	68.4	81.0	82.1	84.0	92.7	76.4
Eat Out	47.9	49.6	66.8	78.6	79.4	81.4	89.9	73.8
Pickup /Drop-off	15.3	16.2	20.4	22.9	23.1	23.5	25.2	21.6
Other Activities	20.8	24.6	30.1	34.8	35.1	35.8	39.2	33.6
Mean Absolute Error	-	10.93	13.78	24.00	24.80	26.48	34.13	20.1

TABLE 5. Out-of-Sample Predictive Performance of MDCEV Time-use Models with Different Approaches for Time Budgets

Observed and predicted activity participation rates								
	Observed	Log-Linear regression	Stochastic frontier	Budget= 875 minutes	Budget= 918 minutes	Budget = 1000 minutes	Budget = 1440 minutes (24 hrs.)	Budget= 24hrs-in home duration
Shopping	64.40%	69.61%	57.91%	55.58%	55.47%	55.31%	55.07%	53.49%
Personal Business	37.30%	49.10%	37.32%	36.03%	36.02%	35.98%	36.10%	34.07%
Social Recreation	35.70%	47.23%	35.16%	33.81%	33.82%	33.80%	33.88%	31.92%
Active Recreation	25.50%	29.74%	22.88%	22.06%	21.99%	21.99%	22.31%	20.81%
Medical	30.00%	34.37%	26.13%	25.25%	25.25%	25.18%	25.24%	23.33%
Eat Out	31.80%	38.60%	29.27%	28.05%	28.04%	27.95%	28.16%	26.43%
Pickup /Drop-off	20.20%	23.94%	17.96%	17.19%	17.21%	17.13%	17.29%	15.92%
Other Activities	8.20%	8.85%	6.67%	6.46%	6.44%	6.43%	6.50%	5.86%
Mean Absolute Error		6.04	2.48	3.58	3.61	3.67	3.57	5.16
Observed and predicted average activity durations (minutes) for those who participated in the activity								
Shopping	53.46	74.66	78.87	93.63	94.80	97.22	108.26	86.48
Personal Business	51.08	57.04	64.21	75.62	76.45	78.29	86.55	71.64
Social Recreation	118.53	102.75	131.34	160.81	162.81	167.36	188.63	149.23
Active Recreation	55.91	28.82	30.48	34.62	35.00	35.64	38.24	32.77
Medical	62.36	59.46	69.55	81.87	82.72	84.83	93.90	77.84
Eat Out	44.67	56.08	67.75	80.32	81.17	83.31	91.82	74.50
Pickup /Drop-off	14.80	16.54	19.83	22.15	22.27	22.67	24.05	21.26
Other Activities	25.66	25.68	29.23	34.59	35.05	35.87	39.32	33.59
Mean Absolute Error		10.76	14.46	24.96	25.70	27.41	34.95	20.89

TABLE 6. Simulated Land-use Impacts on Out-of-Home Time-use Patterns for MDCEV Models with Different Approaches for Time Budgets

MDCEV model with budget from...	Unspent Time	Shopping	Personal Business	Social Recreation	Active Recreation	Medical	Eat Out	Pickup /Drop-off	Other
Log-linear Regression	--	-2.5% (-9)	-1.7% (-8)	2.2% (13)	2.1% (18)	-1.1% (-9)	-1.3% (-8)	-0.6% (-4)	-0.2% (-4)
Stochastic Frontier Regression	-3.6% (-21)	-1.9% (-7)	-1.3% (-7)	3.0% (21)	2.2% (25)	-0.9% (-8)	-1.1% (-7)	-0.3% (-4)	-0.2% (-4)
Budget = 875 minutes	-4.8% (-24)	-1.7% (-7)	-1.1% (-6)	3.9% (21)	2.3% (28)	-0.8% (-7)	-0.9% (-7)	-0.2% (-5)	-0.1% (-4)
Budget = 918 minutes	-4.9% (-24)	-1.6% (-7)	-1.1% (-6)	4.0% (21)	2.3% (28)	-0.8% (-7)	-0.9% (-7)	-0.2% (-5)	-0.1% (-4)
Budget = 1000 minutes	-5.1% (-24)	-1.6% (-7)	-1.0% (-6)	4.1% (21)	2.4% (29)	0.8% (-7)	-0.9% (-7)	-0.2% (-5)	-0.1% (-5)
24hrs-in home duration	-4.3% (-21)	-1.6% (-7)	-1.0% (-6)	3.4% (21)	2.2% (27)	-0.8% (-7)	-0.8% (-7)	-0.2% (-5)	-0.1% (-4)

Note: In each cell, the % number indicates the % of individuals for whom the time allocated to an activity increased or decreased by more than a minute. A positive (negative) number indicates the % of individuals for whom the time allocated to the corresponding activity increased (decreased) by more than a minute. The numbers in the parentheses indicate the average change in the time allocated (minutes) for whom a change occurred in the time allocation to this activity by more than a minute. A positive number indicates an increase in the time allocation while a negative number indicates a decrease in the time allocation. For example, with time budgets estimated using log-linear regression, the MDCEV model predicts that doubling accessibility to recreational land leads to a decrease in the time allocated to shopping by more than a minute for 2.5% of the individuals in the sample. And the average decrease in time allocation to shopping activity for these same individuals is 9 minutes.

Figure 1. Observed and Predicted Activity Durations from MDCEV Models with Different Approaches for Time Budgets

