A JOINT MODEL OF CRASH TYPE AND SEVERITY FOR TWO-VEHICLE CRASHES

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ABSTRACT

This paper presents a joint model of crash type and severity for two-vehicle crashes. Previous work in the area of transportation safety has focused on modeling crash severity as a function of crash type or on modeling crash type alone. However, there have been virtually no studies aimed at modeling the type of collision and level of severity simultaneously. This paper aims to fill this gap in the literature by presenting a joint simultaneous equations model of crash type and severity level using data from the national General Estimates System (GES) in the United States. The simultaneous equations model takes the form of a joint unordered – ordered discrete model where collision type is treated as an unordered multinomial discrete variable while severity is treated as an ordered discrete variable. Model estimation is accomplished using full information maximum likelihood methods (FIML) implemented via simulation approaches. Model estimation results show significant presence of error correlations, i.e., the presence of common unobserved factors that influence crash type and severity, thus supporting the jointness in modeling these two crash characteristics.

Keywords: crash type, crash severity, joint model, unordered-ordered model, unobserved common factors
INTRODUCTION

Crashes may be described by various characteristics including, but not necessarily limited to, the type and number of objects or vehicles involved in the crash, the type of collision, and the severity level. Research in the field of transportation safety has examined factors contributing to the occurrence of a type of crash or collision and severity levels. Models of crash severity often include the type of crash or collision as an explanatory variable; this makes perfect sense as one would expect head-on collisions to result in greater severity levels than rear-end or sideswipe collisions. In other words, crash or collision type is treated as an independent or exogenous variable in models of crash severity. In addition, it is possible to model crash or collision type as a function of roadway characteristics, driver characteristics, vehicle characteristics, and traffic characteristics. Thus, it is possible to model crash type and crash severity level (as a function of crash type) independently and the literature is replete with examples of such model systems (see the next section for references).

This paper aims to model crash type and crash severity jointly through the use of a simultaneous equations system that explicitly accounts for the presence of unobserved factors that may affect both collision type and crash severity. The jointness of the model system explicitly recognizes that crash or collision type may also be endogenous in any model that aims to predict crash severity. Treating crash type as an exogenous variable, when in fact it is endogenous to the phenomenon under study, may lead to inconsistent and biased estimates of coefficients in the model of crash severity. Simultaneity or jointness may arise because there are common unobserved factors that affect both collision type and crash severity (Greene, 2003). For example, there may be some unobserved driver or roadway features and characteristics that are not measurable, but affect the likelihood of both the occurrence of a certain crash type and crash
severity level. For example, driver aggression may be associated with higher likelihood of head-on collisions and more severe crashes, even after controlling for all observed factors.

The collision type is treated as an unordered discrete variable while crash severity is treated as an ordered discrete variable. These definitions are consistent with those routinely found in the literature and constitute an intuitively appealing treatment of these crash descriptors. One of the challenges associated with modeling these two phenomena jointly is that these are two multi-category discrete variables (one unordered and one ordered) and estimating a joint unordered-ordered discrete model system is computationally and analytically complex and burdensome. However, recent advances in discrete choice modeling theory and estimation methods now make it possible to estimate such model systems with relative ease. This paper will therefore provide a unique contribution to the field of transportation safety research by providing a joint model of collision type and severity and offering a method for testing whether there is jointness in these two phenomena.

Data from the National Automotive Sampling System General Estimates System (GES) compiled by the National Highway Traffic Safety Administration’s National Center for Statistics and Analysis in the United States is used in this study. This data set comprises a nationally representative sample of crashes of all types occurring in the United States each year. For this paper, data for the year 2005 are used for analysis and model estimation.

The remainder of this paper is organized as follows. Following a brief discussion on the modeling of crash type and severity, the paper provides a description of the simultaneous equations modeling methodology. This is followed by a description of the data set, presentation of model estimation results, and a discussion of key findings and conclusions that can be drawn from the model results.
ANALYZING CRASH TYPE AND SEVERITY

The analysis of crash type and severity level has been the subject of much study in the transportation safety arena for many years. It is not possible to provide a comprehensive review of the literature on these topics within the scope of this paper; however, even a cursory review of the literature serves as evidence of the importance of and interest in modeling collision type and crash severity with a view to devise measures and adopt designs that can mitigate both of these crash characteristics.

The literature on transportation safety is replete with examples of discrete choice models for estimating crash severity. Abdel-Aty (2003) developed ordered probit models to analyze factors affecting injury severity at roadway sections, signalized intersections and toll plazas. The study identified driver’s age, gender, seat belt use, point of impact, speed, and vehicle type as major factors affecting crash severity. Abdel-Aty and Keller (2005) developed an ordered probit model to study the effects of different factors on injury severity for crashes at intersections. Kockelman and Kweon (2002) used an ordered probit methodology to estimate an injury severity model for total crashes, single vehicle crashes and two vehicle crashes separately using data from the 1998 National Automotive Sampling System GES. Chang and Mannering (1999) used a nested logit framework to identify the relationships between injury severity and vehicle occupancy. They developed separate nested logit models for crashes involving trucks and crashes not involving trucks. A discrete choice modeling framework that does not consider ordinality of injury severity was developed by Shankar and Mannering (1996). They used a multinomial logit model to study crash severity involving a motorcycle. Ulfarsson and Mannering (2004) estimated multivariate multinomial logit models to analyze injury severity separately for males and females.

The logistic regression methodology has also been used to estimate crash prediction models. Farmer et al. (1996) used logistic regression framework to study the effect of crash,
vehicle, driver, and occupant characteristics on the injury type and severity in two-vehicle side impact crashes. Al-Ghamadi (2002) also estimated a logistic regression model of injury severity and identified location and cause of crash as the two important variables explaining crash severity. Bédard et al. (2002) developed a multivariate logistic regression model to describe the effect of driver characteristics, crash characteristics, and vehicle characteristics on fatal crashes. Shibata and Fukuda (1993) examined the possible risk factors that lead to fatalities by estimating a multiple logistic regression model. Huang et al. (2007) considered the driver-vehicle correlations and developed a Bayesian hierarchical binomial logistic model to estimate crash severity and vehicle damage in intersection crashes. Dissanayake and Lu (2002), Toy and Hammit (2003), and Conroy et al. (2006) constitute additional examples of the application of logistic regression models for analyzing crash severity. Other notable works that used discrete choice models include Shankar et al (1996), O’Donnell and Connor (1996), Renski et al. (1999), Srinivasan (2002), Gårder (2006). More recently, Milton et al. (2007) developed a mixed logit model of crash severity to capture the unobserved heterogeneity in observed variable effects on crash severity.

Other modeling approaches to study crash severity include Kattak et al. (2002) who used poisson and negative bionomial models to estimate rollover intensity and modeled injury severity using a weighted ordered logit model, Kweon et al. (2003) who combined exposure data (vehicle miles driven) with crash rates and injury severity to assess the risk to different driver groups, Santamarihna-Rubio et al. (2007) who conducted a cross-sectional study to examine the injury profiles across various roadway user groups with specific emphasis on fatalities, and Boufous et al. (2008) who used univariate and multivariate linear regression techniques to identify environmental, vehicle, crash, and driver characteristics that impact injury severity in older drivers.

Much of the previous research in the safety arena is devoted to the development of crash prediction models by injury severity. In contrast, there has been little work on the development of
models to predict the type of crash. Golob et al. (1986) present one of the few models that can predict crash frequency by crash type. They used a log-linear model to study the effect of collision type and crash characteristics on injury severity in crashes involving trucks. Kim et al. (1994) also used a log-linear modeling approach to examine the effect of driver characteristics and behaviors on crash type and injury severity. McCartt et al. (2004) identified the most common crash types occurring at heavily traveled urban interstate ramps in Northern Virginia using illustrations and descriptions from police crash reports. More recently, Braitman et al. (2008) studied non-fatal crashes involving 16 year old drivers using data from Connecticut. The study examined police crash reports and interviews to identify the major crash types and the contributing factors.

In much of the research highlighted here, injury severity is modeled while considering the nature and type of crash as exogenous to the model. In other words, the possibility that crash or collision type might be endogenous to the system is largely ignored. However, more recently, there has been an attempt to develop joint models that can account for endogeneity in transportation safety studies. Kim and Washington (2006) developed a joint model to account for endogeneity of left lane presence in angle crashes at intersections. They used Limited Information Maximum Likelihood (LIML) estimation methods to estimate their model system.

Recent advances in Full Information Maximum Likelihood (FIML) estimation methods have begun to be exploited in joint models of safety outcomes. For example, Eluru and Bhat (2007) developed a joint ordered-unordered modeling framework to analyze seat belt usage and injury severity. It should be noted that the joint modeling framework presented by Eluru and Bhat (2007) is different from that proposed and applied earlier in a travel behavior modeling context (Bhat, 1997). The earlier version involves the use of a functional transformation to obtain a closed form likelihood function. The latest version of the joint ordered-unordered model is formulated based on more flexible assumptions on the error distribution, but the likelihood function does not
take a closed form. However, recent advances in simulation-based estimation approaches allow one to approximate functions that do not take a closed form.

The modeling methodology that is adopted in this paper may be considered an extension of the joint modeling framework presented by Eluru and Bhat (2007). The difference is that while their work involved the joint analysis of a binary outcome and multinomial ordered outcome variable, this paper involves the joint analysis of a multinomial unordered variable and ordered multinomial variable. This is because crash (collision) type is treated as a multinomial unordered outcome, while injury severity is treated as a multinomial ordered outcome. It should be noted that the extension from the case of binary outcomes to that of multinomial outcomes is not straightforward from a methodological perspective. The following section presents details on the modeling methodology.

MODELING METHODOLOGY

This section presents the modeling methodology adopted in this paper for jointly modeling crash or collision type and crash severity level. As mentioned earlier, collision type is treated as an unordered discrete variable while crash severity is treated as an ordered discrete variable. Therefore, a joint model system of unordered and ordered discrete endogenous variables is developed and presented in this paper. The joint model system may be written as follows:

\[
\begin{align*}
U_{ji}^* &= x_{ji} \beta_j + \xi_j n_{ji} + \epsilon_{ji} \\
V_i^* &= z_i \gamma + \sum_{t=1}^{J-1} \delta_t y_{ti} + \sum_{k=1}^J \eta_k n_{ki} + \mu_i
\end{align*}
\]

where

- \(i\) is an index denoting observations;
- \(j = 1, 2, \ldots, J\), where \(J\) is the number of possible outcomes in unordered discrete variable;
- \(U_{ji}^*\) is latent propensity function for unordered outcome \(j\) and observation \(i\);
$V_i^*$ is latent propensity function for ordered outcome of observation $i$;

$x_{ji}$ is a vector of explanatory variables in $U_{ji}^*$ and $\beta_j$ is a vector of corresponding coefficients;

$z_i$ is a vector of explanatory variable in $V_i^*$ and $\gamma$ is a vector of corresponding coefficients;

$y_{ti}$ is a series of dummy variables indicating whether outcome $t$ occurs for observation $i$ and $\delta_j$ is the coefficient corresponding to $y_{ti}$ ($y_{ti} = 1$ as long as $U_{ti}^*$ is the maximum among all $U_{ji}^*$. $y_{ti}$ are generated based on values of $U_{ji}^*$ and therefore constitute endogenous variables in $V_i^*$. This calls for the use of a joint simultaneous equations modeling system for consistently estimating $\delta_j$);

$\varepsilon_{ji}$ is an idiosyncratic random component in $U_{ji}^*$, which is assumed to be i.i.d. standard Gumbel distributed;

$\mu_i$ is an idiosyncratic random component in $V_i^*$, which is assumed to be standard normally distributed;

$n_{ji}$ is a random variable representing heterogeneity in $U_{ji}^*$, which is assumed to be standard normally distributed and its coefficient $\xi_j$ constitutes its standard deviation;

all $n_{ji}$ are specified in $V_i^*$ as a linear combination with coefficients $\eta_k$, which provides for accommodating correlation between each pair of $U_{ji}^*$ and $V_i^*$.

The correlation between $U_{ji}^*$ and $V_i^*$ can be expressed as:

$$
\text{Corr} (U_{ji}^*, V_i^*) = \frac{\xi_j \eta_j}{\sqrt{\left(\xi_j^2 + \frac{\pi^2}{6}\right) \sum_k \eta_k^2 + 1}}
$$

$V_i^*$ is the latent propensity for crash severity and it can be scaled up or down without changing the probability of crash severity outcomes. As a result, the standard deviation $\eta_j$ of the error component cannot be uniquely identified unless certain parameter restrictions are placed. One
may initially fix $\eta_j$ to be equal to 1 for identification purposes and the correlation can then be expressed as:

$$Corr(U_{ji}^*, V_i^*) = \frac{\xi_j}{\sqrt{\left(\xi_j^2 + \frac{\pi^2}{6}\right)(J+1)}}$$

(3)

One limitation of Equation (3) is that it cannot accommodate any level of inter-propensity correlation because $\lim_{\xi_j \to \pm \infty} \frac{\xi_j}{\sqrt{\left(\xi_j^2 + \frac{\pi^2}{6}\right)(J+1)}} = \pm \frac{1}{\sqrt{J+1}}$ as $\xi_j$ approaches positive or negative infinity. If $J = 5$, this limit is equal to 0.4082 or –0.4082. In other words, the inter-propensity error correlation cannot fall outside the range of −0.4082 to +0.4082 when $\eta_j$ is constrained to be equal to 1. To overcome this limitation, it is assumed that the absolute value of $\eta_j$ is equal to $\xi_j$ and the sign of $\eta_j$ is the same as that of the estimates of $\xi_j$ obtained when $\eta_j$ is fixed to be equal to 1. With such identification restrictions, one obtains:

$$Corr \left( U_{ji}^*, V_i^* \right) = \frac{\pm \xi_j^2}{\sqrt{\left(\xi_j^2 + \frac{\pi^2}{6}\right) \left( \sum_{k=1}^{i} \xi_k^2 + 1 \right)}}$$

(4)

These identification restrictions are consistent with those used by Eluru and Bhat (2007) and can accommodate any level of error correlation ranging from -1 to 1.

For model estimation, the probability function needs to be formulated for deriving the likelihood function. Conditional on common normal heterogeneity $n_{ji}$, the modeling system consists of one multinomial logit model representing the unordered discrete crash type outcomes and one standard ordered probit model representing ordered discrete crash severity levels. Thus, it is straightforward to derive the conditional probability function for unordered outcomes as:
For ordered outcomes, let $w_i$ be a variable indicating ordered outcome for observation $i$ and $\alpha_p$ be a series of ordered constant thresholds that help determine the ordered outcome based on the rule that, if $\alpha_{p-1} < V_i^* < \alpha_p$, then $w_i$ is equal to $p$ ($p = 1, 2, \ldots, P$; $P =$ highest possible ordered outcome such as a fatality). Among all the $\alpha_p$, $\alpha_0 = -\infty$ and $\alpha_P = +\infty$; therefore there are $(P-1)$ thresholds to be estimated in the model. It should be noted that there is no column of ones in vector $z_i$ denoting the constant term in the propensity function because threshold $\alpha_1$ has been left free to be estimated (as opposed to constraining it to a value of zero). Then, the conditional probability function for ordered outcome $w_i$ can be formulated as:

$$\Pr(w_i = p \mid n_{ki}) = \Phi(\alpha_p - z_iy_i - \sum_{k=1}^{j-1} \delta_i n_{ki}) - \Phi(\alpha_{p-1} - z_iy_i - \sum_{k=1}^{j-1} \delta_i n_{ki})$$

(6)

To obtain the unconditional probability function, one can integrate $n_{ki}$ over their distributional domain as:

$$\Pr(y_{ji}, w_j) = \int \Pr(y_{ji} \mid n_{ki}) \Pr(w_{pi} \mid n_{ki}) f(n_{ki}) \, dn_{ki}$$

(7)

Note that Equation (7) constitutes a multidimensional integral consistent with the dimension of $n_{ki}$.

The unconditional probability function does not have a closed form solution and hence the Monte Carlo simulation method is adopted to approximate $\Pr(y_{ji}, w_{pi})$ by its simulated probability (SP) as:

$$SP(y_{ji}, w_j) = \frac{1}{R} \sum_{r=1}^{R} \Pr(y_{ji} \mid n_{kir}) \Pr(w_{pi} \mid n_{kir}),$$

(8)

where $n_{kir}$ are $J$ sets of independent random seeds drawn from a standard normal distribution for approximating $SP(y_{ji}, w_{pi})$. Then, the Maximum Simulated Likelihood Estimation (MSLE)
method can be applied to estimate all of the unknown parameters including $\beta_j$, $\gamma$, $\alpha_p$, $\delta_i$, $\xi_j$, and $\eta_k$ by maximizing the following log-likelihood function:

$$LL = \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{p=1}^{P} \left\{ I(i, j, p) \ln[SP(y_{ji}, w_i)] \right\},$$

where $I(i, j, p) = 1$ if $y_{ji} = 1$ and $w_i = p$; $I(i, j, p) = 0$, otherwise.

For estimation purposes, 200 Halton sequences (Bhat, 2001) are drawn for approximating the log-likelihood function. The log-likelihood function and its first-order derivative are coded in Gauss (Aptech, 2006) and the Maxlik module is employed to estimate parameters that maximize the log-likelihood function.

**DATA PREPARATION AND DESCRIPTION**

The data used in this paper is derived from the National Automotive Sampling System General Estimates System (GES) compiled by the National Highway Traffic Safety Administration’s National Center for Statistics and Analysis. GES serves as a nationally representative probability sample randomly selected from about six million police-reported crashes in 60 geographic sites across the United States. The data collectors make regular visits to approximately 400 police agencies within the 60 sites and compile a list of all qualifying crashes reported since their last visit. A sample of these crashes is selected for inclusion in the nationally representative database.

GES primarily consists of three datasets: Accident, Vehicle and Person. The Accident dataset, in which each record represents one crash, contains information on crash characteristics and environmental conditions at the time of the crash. The Vehicle dataset provides information on the vehicles involved in the crash, with each record providing information on one involved vehicle. The Person dataset provides information on all persons involved in the crash including drivers, passengers, pedestrians, cyclists, and others, with each record representing one person.
The analysis in this paper utilizes the most updated 2005 GES data, which contains detailed information for 54,597 crashes involving 96,340 vehicles and 137,884 persons. As this paper involves jointly modeling collision type and severity level, it was felt that the analysis should be limited to two-vehicle crashes. The collision types associated with single-vehicle and multi-vehicle (greater than two vehicles) crashes may be very different from those seen in two-vehicle crashes. Therefore, to control for variability in collision type definitions, only two-vehicle crashes are considered in the modeling effort reported in this paper. After cleaning the data, 30,757 two-vehicle crashes with valid severity information were selected for analysis.

Table 1 provides a descriptive analysis of selected variables available in the dataset. Most variables, except vehicle age, vehicle occupancy, and speed limit, are dummy variables that take a value of zero or one. The table furnishes mean and standard deviation values for the variables. The mean values indicate, for example, that alcohol is involved in 6.1 percent of two-vehicle crashes, dark light conditions existed at the time of 6.3 percent of crashes, and so on. Most notably in the context of this paper is that nearly one-half of the crashes are angle crashes and about one-third are rear-end crashes. About 10 percent are sideswipes while just about five percent are head-on collisions. The subject driver is female in 43.3 percent of crashes. The subject driver is the driver in the vehicle that suffered the higher level of severity. The subject vehicle is randomly assigned if both vehicles experienced the same level of crash severity. The subject driver was wearing a seatbelt in 77 percent of the crashes. Similarly, descriptive statistics can be noted for the other variables listed in the table.

*(Insert Table 1 here)*

Table 2 provides a cross-tabulation of crash severity by crash (collision) type. Crash severity is represented by five categories: no injury (property damage only), possible injury, non-incapacitating (minor) injury, incapacitating (severe) injury, and fatal. Crash type is characterized...
by five categories as well: rear-end, head-on, angle, sideswipe, and other. The cross-tabulation sheds light on the inter-relationship between crash type and severity level. The first block of the table provides frequencies while the second and third blocks of the table provide row and column percents respectively. An examination of the row percents shows that the percent of property damage only crashes that are rear-end type is nearly 40 percent; this percent decreases steadily with increasing severity level until it reaches just about 15 percent for fatal crashes. On the other hand, the percent of property damage only crashes that are head-on is just about 2.3 percent and this percentage consistently increases with rising severity level until it reaches nearly 19 percent for fatal crashes. Nearly one-half of all crashes in all severity categories are angle crashes. About 12 percent of fatal crashes are sideswipes. These trends are further seen in the block of the table providing column percents. While more than one-half of rear-end crashes result in no injury, only 22 percent of head-on collisions do so. While less than one percent of rear-end collisions end in fatality, about 6 percent of head-on collisions do so. Thus there is a clear relationship between collision type and crash severity with head-on collisions being the most prone to leading to serious injury or a fatality relative to other crash types. This study aims to better understand the relationship between these two crash descriptors while explicitly accounting for simultaneity that may arise due to the presence of common unobserved factors that influence both crash type and severity level.

*(Insert Table 2 here)*

**MODEL ESTIMATION RESULTS**

Model estimation results are presented in Tables 3 and 4. Table 3 shows the model estimation results for the crash type model (unordered discrete outcomes model) while Table 4 shows the results for the ordered crash severity component of the model system. As mentioned in the
modeling methodology section, model estimation is performed in two stages. First, a preliminary model estimation is performed assuming that all $\eta_j$ are equal to one. This model estimation result is presented in the left columns of the tables. This preliminary estimation aids in identifying the signs of error correlations between each pair of propensity functions for ordered and unordered outcomes. According to the identified signs (see signs for the $\xi$ parameters in the table) and imposing the identification restrictions that $|\xi_j| = |\eta_j|$, final model estimation results are obtained and presented in the middle block of the tables. Tables 3 and 4 also include estimation results for a system of independent crash type and severity models. For this model system, $\xi_j$ and $\eta_j$ are fixed at zero, i.e., no error correlation is accommodated in the model system although crash type is included as an explanatory (exogenous) variable in the severity model. In the absence of correlations, the joint model reduces to a set of two independent models – a multinomial logit model of crash type and an ordered probit model of crash severity, similar to those most encountered in the research literature. The estimation results for these independent models are provided in the last two columns of Tables 3 and 4 respectively.

(Insert Tables 3 and 4 here)

Model estimation results presented in Table 3 suggest that numerous exogenous variables influence the type of collision that occurs. The constant for head-on collision type is the lowest among all collision types suggesting that this collision type is the least likely to occur relative to other collision types. Conversely, angle collisions are the most likely to occur, followed by rear-end collisions as evidenced by the magnitudes of the constants associated with each outcome propensity function. The presence of a curve decreases the likelihood of a rear-end collision and a sideswipe collision, but increases the likelihood of a head-on collision, possibly due to blind curves (poor sight distance) and difficulty in vehicle maneuverability. As expected, stop and yield
sign traffic controls result in a higher likelihood of rear-end collisions; when vehicles stop at these traffic controls, vehicles that come from behind may strike the vehicle in front if the following driver failed to stop in time. These traffic controls also contribute to higher rates of angle crashes, presumably because drivers make errors in judgment at these locations which are often characterized by vehicular conflicts. Head-on collisions are less likely to occur during the day due to better visibility during the day. Rainy weather increases the chance of rear-end collisions while icy surface conditions increase the likelihood of head-on collisions, presumably because vehicles have difficulty in stopping or avoiding the collision under such conditions. Alcohol involvement is positively associated with head-on collision occurrence. The \( f \) values shown in the table represent the unobserved heterogeneity in crash type outcomes. It appears that this parameter is statistically significant only for rear-end collisions and head-on collisions. Overall, the model estimation results clearly show that a host of roadway geometry variables, driver characteristics, vehicle characteristics, and environmental conditions affect collision type, thus indicating that collision type is a dependent variable itself and is not best treated as an exogenous or independent variable.

Table 4 presents the ordered outcomes model of crash severity. Once again, a two-step estimation approach is adopted and the final model is shown in the middle block of the table. It is interesting to note that wet, icy, and snow roadway surface conditions are associated with lower severity levels. This may be due to the fact that drivers are driving more carefully and at slower speeds under these conditions. As such, the severity of crashes that occur under these conditions is reduced, even if the potential for crash occurrence is greater. As the speed limit increases, the severity of crashes increases; conditional upon a crash occurring, one would expect crash severity to be higher at higher speeds. The presence of a yield sign results in lower severity levels, again presumably due to slower speeds at such locations. Severity levels increase as a function of
numerous variables of interest. Severity levels rise when it is dark and when alcohol is involved. If the subject driver is a female, severity levels are likely to rise. It is possible that females are more prone to bodily injury and death when involved in a crash due to their weaker physical constitution and potentially different reaction when involved in a crash (Ulfarsson and Mannering, 2004). As the subject vehicle occupancy increases, severity levels rise.

Severity levels are also higher for older vehicles that may not be equipped with the same level of safety equipment as newer vehicles. Severity levels are highest when the subject vehicle is a standard 4-door hardtop sedan closely followed by 2-door hardtop sedans. Severity levels are likely to be lower for medium/heavy trucks and utility vehicles; this finding is consistent with the notion that larger and heavier vehicles better protect the occupants of the vehicle relative to standard 4-door and 2-door cars. However, among all vehicle type categories, the severity level is likely to be the highest for motorcycle riders. Severity levels rise substantially when the driver or passengers are ejected, possibly through a windshield. Seatbelt use is associated with lower levels of crash severity.

More notable in the context of this study is the finding that the collision type variable is found to significantly impact crash severity. Severity level is likely to be the highest for head-on collisions, followed by angle crashes. Lower severity levels are associated with rear-end and sideswipe collisions as evidenced by the statistically significant negative coefficients in the model. In summary, all of the model estimation results are intuitively appealing and reasonable. The threshold coefficients are all reasonable and statistically significant indicating that the ordered response representation of crash severity is appropriate in the context of this data set. The goodness-of-fit statistics (represented by the $\rho^2$ term and the difference between the log-likelihood function value at 0, $L(0)$, and at the estimated model coefficients, $L(\beta)$, indicate a reasonable model fit, consistent with what one might expect for a model of this nature.
Finally, the model estimation results clearly show that there is significant error correlation between selected crash (collision) type propensities and crash severity. The error correlations are calculated based on the $\xi_i$ values using Equation (4) and their statistical significance can also be determined based on the values of $\xi_i$. It is found that the first two error correlation terms with absolute values greater than 0.1 are statistically significant while the other three are not statistically significant at the 95 percent confidence level.

The first significant error correlation is that between the propensities for rear-end collision and crash severity and is positive. The second one is that between the propensities for head-on collision and crash severity and is negative. The finding of significant error correlations between two propensities of crash type and crash severity supports the adoption of and calls for the use of a simultaneous equations approach to jointly model collision type and crash severity. The presence of significant error correlations suggests that there are common unobserved factors that are simultaneously affecting these two crash type outcomes and crash severity. The signs of these error correlations are opposite to that associated with the coefficient of the corresponding endogenous variable category included in the severity model. For example, the coefficient of rear-end collision type (endogenous variable) in the severity model is negative; the error correlation is then positive. The opposite holds true for the case of the head-on collision type outcome. This result is consistent with that reported in the literature (Eluru and Bhat, 2007). Essentially, one can interpret this finding as follows. Unobserved factors contributing to a rear-end collision are positively correlated with unobserved factors contributing to crash severity. On the flipside, unobserved factors contributing to a head-on collision are negatively correlated with unobserved factors contributing to crash severity.

To illustrate this, consider that poor vehicular maintenance is likely to be an unobserved factor contributing to crash severity. An unobserved factor contributing to a rear-end collision
may be poor brakes or braking ability of the vehicle. As poor vehicle maintenance is correlated with poor condition of the brakes, there is a positive correlation between the unobserved factors influencing rear-end collision outcome and crash severity. Now consider that an unobserved factor contributing to a head-on collision may be excessive speed of travel in a high-performance vehicle. However, it is unlikely that a driver using a poorly maintained vehicle will travel at excessive speeds and it is unlikely that a high-performance vehicle will be poorly maintained. In other words, in this instance, there is a negative correlation between the unobserved factor affecting severity (poorly maintained vehicle) and head-on collision occurrence (aggressive driving at high speeds in a high-performance vehicle).

A comparison between estimation results of the joint model and independent model offers insights into the importance and impact of accounting for error correlations (i.e., jointly modeling crash type and severity). It is to be noted that the coefficients of exogenous variables in the ordered model will be scaled up by the standard deviation of the random error component. In other words, coefficients in the ordered probit model that accounts for endogeneity will be scaled up by a factor equal to \( \sqrt{\sum \eta_j + 1} \approx 1.34 \), relative to those in the independent ordered probit model. This is generally found to hold true for coefficients of exogenous variables. However, a difference is seen in the context of the endogenous variables representing crash type included in the severity model. In this particular model estimation effort, it is found that the ratio of coefficients of endogenous dummy variables for rear-end and head-on crashes (between the joint and independent ordered probit models) are 1.68 and 2.33 respectively. These ratios are substantially higher than the ratio of 1.34 that would have been realized had there been no endogeneity bias. This implies that an independent ordered probit model of severity, which does not account for endogeneity of crash type, considerably underestimates the severity of both rear-end crashes and head-on crashes.
However, between these two crash types, the severity of head-on crashes is underestimated by a substantially larger degree than that of rear-end crashes. In addition, a comparison of the adjusted likelihood ratio indices between the two models (0.2660 for the joint model versus 0.2654 for the independent model) suggests that accounting for endogeneity offers some gains in goodness of fit. These findings suggest that it is important to jointly model crash type and severity and account for endogeneity bias that may arise if one were to treat crash type as a purely exogenous variable in severity models.

**CONCLUSIONS**

This paper presents a simultaneous equations joint model of crash type (collision type) and crash severity using a nationally representative sample of two-vehicle crashes obtained from the General Estimates System (GES) maintained by the National Highway Traffic Safety Administration (NHTSA) in the United States. Crash type or collision type is characterized by five different collision possibilities: rear-end, head-on, angle, sideswipe, and other. Crash severity is defined by five different levels: property damage only or no injury, possible injury, non-incapacitating injury, incapacitating injury, and fatality. The collision type is treated as a multinomial unordered discrete outcome variable while the crash severity is treated as an ordered discrete response variable. A joint unordered – ordered discrete outcome model system is formulated and the simultaneity in the phenomena is tested through the inclusion of error components that capture the presence of common unobserved factors that influence crash type and severity.

Model estimation results are found to be plausible and intuitively reasonable. Numerous exogenous variables including roadway surface condition, environmental conditions, traffic control devices, speed limit, vehicle occupancy and vehicle age, alcohol involvement, driver gender, roadway alignment, and vehicle body type were significant in explaining either crash type
or crash severity or both. It was found that crash type significantly affected severity level with head-on collisions associated with the highest severity levels, consistent with expectations. More important in the context of this study is the finding that two of the five error correlations were statistically significant at the 95 percent confidence level. The presence of significant error correlations strongly suggests that there are common unobserved factors affecting collision type and crash severity. Therefore, crash type and severity should be modeled jointly in a simultaneous equations system; not doing so (i.e., treating crash type as an exogenous variable in crash severity models) may result in inconsistent and biased coefficient estimates in single equation severity models.

This paper makes both methodological and empirical contributions to the study of transportation safety. From a methodological standpoint, the paper has presented a joint unordered-ordered discrete outcome model system that is capable of simultaneously modeling crash type and ordered severity while accounting for error correlations. The paper has presented an econometrically rigorous framework for estimating such a model system using new simulated maximum likelihood estimation methods. The paper also includes a discussion on the identification restrictions that need to be placed on the model system so that parameter values can be estimated. From an empirical standpoint, the paper presents model estimation results that show the impacts of numerous exogenous variables on crash type and severity and the impact of crash type (an endogenous variable) on severity. The empirical results show the simultaneity or jointness in crash type and severity (at least in the context of this data set) and chart a new direction for modeling of crash type and severity in the future. Future research should examine whether the finding of simultaneity holds true across crash and location contexts and should focus on identifying the unobserved factors that may be contributing to the simultaneity, so that data on such unobserved factors may be collected and recorded if feasible.
REFERENCES


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Table 2. Descriptive Analysis of Dependent Variables

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<th>Angle</th>
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Row Percent

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<th>Fatal</th>
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Column Percent

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<td>100.0%</td>
<td>100.0%</td>
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Table 3. Unordered Multinomial Logit Model of Crash (Collision) Type

| Variables                              | $\eta_j = 1$ | $|\eta_j| = |\xi_j|$ | $\eta_j = 0$ |
|----------------------------------------|--------------|-----------------|--------------|
|                                        | Coef         | t-stat          | Coef         | t-stat          | Coef         | t-stat          |
| Rear-End Collision Propensity Function |              |                 |              |                 |              |                 |
| Constant                               | 2.4569       | 43.458          | 2.4490       | 45.633          | 2.6454       | 64.603          |
| Traffic Control Device: Stop Sign      | 0.5529       | 4.571           | 0.2530       | 2.782           | 0.2859       | 3.254           |
| Weather: Rain                          | 0.1844       | 4.258           | 0.1825       | 2.782           | 0.2859       | 3.254           |
| $\xi_1$                                | 0.2310       | 1.546           | 0.3579       | 3.155           | 0.0000       | --              |
| Head-On Collision Propensity Function  |              |                 |              |                 |              |                 |
| Constant                               | -2.4479      | -3.135          | 0.4039       | 3.151           | 0.7208       | 10.961          |
| Light Condition: Daylight              | -0.4464      | -3.608          | -0.2173      | -3.390          | -0.2880      | -4.819          |
| Alcohol Involved                       | 1.3878       | 5.618           | 0.7303       | 7.808           | 0.8472       | 10.089          |
| Roadway Alignment: Curve               | 1.1193       | 3.872           | 0.2951       | 2.374           | 0.2294       | 1.916           |
| Surface Condition: Ice                 | 0.9658       | 2.734           | 0.6268       | 3.326           | 0.6032       | 3.420           |
| $\xi_2$                                | -3.2730      | -5.986          | -0.7907      | -4.551          | 0.0000       | --              |
| Angle Collision Propensity Function    |              |                 |              |                 |              |                 |
| Constant                               | 2.8147       | 59.615          | 2.8211       | 67.115          | 2.8019       | 69.074          |
| Roadway Alignment: Curve               | -1.1354      | -10.621         | -1.1347      | -10.621         | -1.1371      | -10.671         |
| Traffic Control Device: Stop Sign      | 2.5891       | 23.183          | 2.3143       | 28.928          | 2.2884       | 30.089          |
| Traffic Control Device: Yield Sign     | 1.0110       | 4.645           | 0.9326       | 4.622           | 0.9222       | 4.634           |
| $\xi_3$                                | -0.0906      | -0.677          | -0.1529      | -1.004          | 0.0000       | --              |
| Sideswipe Collision Propensity Function|              |                 |              |                 |              |                 |
| Constant                               | 1.4707       | 22.541          | 1.4988       | 30.254          | 1.5096       | 34.797          |
| Roadway Alignment: Curve               | -0.3044      | -2.679          | -0.3122      | -2.764          | -0.3191      | -2.832          |
| $\xi_4$                                | -0.3284      | -1.958          | -0.1725      | -1.064          | 0.0000       | --              |
| Other Collision Propensity Function    |              |                 |              |                 |              |                 |
| Surface Condition: Ice                 | 0.8504       | 3.985           | 0.8465       | 3.968           | 0.8503       | 3.985           |
| $\xi_5$                                | 0.0757       | 0.225           | 0.0467       | 0.227           | 0.0000       | --              |
Table 4. Ordered Probit Model of Crash Severity

| Variables                                      | $\eta = 1$ | $|\eta| = |\xi|$ | $\eta = 0$ |
|-----------------------------------------------|------------|----------------|------------|
|                                               | Coef       | t-stat         | Coef       | t-stat         | Coef     | t-stat         |
| Roadway Surface Condition: Wet                | -0.1420    | -3.248         | -0.0833    | -3.289         | -0.0636  | -3.496         |
| Roadway Surface Condition: Ice                | -0.8746    | -6.107         | -0.4541    | -5.052         | -0.3186  | -5.404         |
| Roadway Surface Condition: Snow               | -0.9239    | -8.429         | -0.5196    | -6.949         | -0.3859  | -8.401         |
| Traffic Control Device: Speed Limit           | 0.0350     | 25.250         | 0.0194     | 11.174         | 0.0145   | 25.027         |
| Traffic Control Device: Yield Sign            | -0.6119    | -4.133         | -0.3763    | -4.422         | -0.3230  | -5.440         |
| Light Condition: Dark                         | 0.3532     | 5.501          | 0.1976     | 5.213          | 0.1593   | 5.975          |
| Alcohol Involved                              | 0.9226     | 14.339         | 0.5518     | 11.036         | 0.4364   | 16.498         |
| Driver being Female\(^b\)                     | 0.3629     | 11.093         | 0.1995     | 8.156          | 0.1484   | 10.873         |
| Vehicle Occupancy\(^b\)                       | 0.2744     | 21.743         | 0.1680     | 11.595         | 0.1285   | 20.655         |
| Vehicle Age (Years)\(^b\)                     | 0.0294     | 10.533         | 0.0161     | 7.877          | 0.0120   | 10.174         |
| Utility Vehicle\(^b\)                         | 0.1113     | 2.143          | 0.0632     | 2.145          | 0.0464   | 2.145          |
| 2-door Sedan, Hardtop, Coupe\(^b\)           | 0.3330     | 5.412          | 0.1838     | 4.931          | 0.1379   | 5.372          |
| 4-door Sedan, Hardtop\(^b\)                   | 0.4276     | 11.491         | 0.2336     | 8.332          | 0.1746   | 11.221         |
| Medium/Heavy Trucks (> 4,536 kg)\(^b\)       | -1.7519    | -22.037        | -1.0119    | -10.930        | -0.7568  | -22.302        |
| Motorcycle\(^b\)                              | 2.7780     | 29.475         | 1.5587     | 11.474         | 1.1656   | 29.793         |
| Driver/Passengers Ejected\(^b\)              | 4.2880     | 15.117         | 2.4383     | 9.803          | 1.8291   | 15.426         |
| Driver Using Seatbelt\(^b\)                   | -0.3537    | -8.934         | -0.1883    | -7.036         | -0.1407  | -8.486         |
| Accident type: Rear-End                       | -0.8915    | -2.623         | -0.5527    | -5.373         | -0.3274  | -7.812         |
| Accident type: Head-On                        | 3.3520     | 10.072         | 1.3975     | 4.377          | 0.5993   | 12.159         |
| Accident type: Angle                          | 0.3340     | 1.044          | 0.1027     | 1.455          | 0.0572   | 1.372          |
| Accident type: Sideswipe                     | -0.7781    | -2.140         | -0.6421    | -6.757         | -0.5049  | -11.192        |

\(\eta_1\) 1.0000 -- 0.3579 3.155 0.0000 --
\(\eta_2\) 1.0000 -- 0.7907 4.551 0.0000 --
\(\eta_3\) 1.0000 -- 0.1529 1.004 0.0000 --
\(\eta_4\) 1.0000 -- 0.1725 1.064 0.0000 --
\(\eta_5\) 1.0000 -- 0.0467 0.227 0.0000 --

Threshold Coefficients

\(\alpha_1\) 1.9029 5.867 0.9865 8.018 0.7376 13.505
\(\alpha_2\) 3.2576 10.040 1.7407 10.071 1.3017 23.740
\(\alpha_3\) 4.8313 14.876 2.6182 11.029 1.9577 35.442
\(\alpha_4\) 7.8722 24.124 4.3465 11.717 3.2500 55.540

Sample Size 30757 30757 30757
Number of Parameters 47 47 42
\(L(\beta)\) -72682.48 -72620.05 -72684.33
\(L(0)\) -99002.96 -99002.96 -99002.96
Adj. \(\rho^2\) 0.2654 0.2660 0.2654
\(\rho_{11}^e\) (Rear-End Collision and Severity) 0.0724 0.1097 0.0000
\(\rho_{22}^e\) (Head-on Collision and Severity) -0.3801 -0.2142 0.0000
\(\rho_{33}^e\) (Angle Collision and Severity) -0.0288 -0.0483 0.0000
\(\rho_{44}^e\) (Side-swipe Collision and Severity) -0.1013 -0.0544 0.0000
\(\rho_{55}^e\) (Other Collision and Severity) 0.0241 0.0149 0.0000

\(^a\)statistically significant at the 95 percent confidence level
\(^b\)variables related to subject vehicle