Accounting For Multi-dimensional Dependencies Among Decision-makers Within a Generalized Model Framework: An Application to Understanding Shared Mobility Service Usage Levels

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Submitted for Presentation Only
97th Annual Meeting of the Transportation Research Board

Committee on Traveler Behavior and Values (ADB10)

Word count: 6480 text + 4 tables/figures x 250 = 7480 words
August 2017

ABSTRACT
Activity-travel choices of individuals are influenced by spatial dependency effects. As individuals interact and exchange information with, or observe the behaviors of, those in close proximity of themselves, they are likely to shape their behavioral choices accordingly. For this reason, econometric choice models that account for spatial dependency effects have been developed and applied in a number of fields, including transportation. However, spatial dependence models to date have largely defined the strength of association across behavioral units based on spatial or geographic proximity. In the current context of social media platforms and ubiquitous internet and mobile connectivity, the strength of associations among individuals is no longer solely dependent on spatial proximity. Rather, the strength of associations among individuals may be based on shared attitudes and preferences as well. In other words, behavioral choice models may benefit from defining dependency effects using attitudinal constructs in addition to geographical constructs. In this paper, frequency of usage of car-sharing and ride-hailing services is modeled using a generalized heterogeneous data model (GHDM) framework that incorporates multi-dimensional dependencies among decision-makers. The model system is estimated on the 2014-2015 Puget Sound Regional Travel Study survey sample, with proximity in latent attitudinal constructs defined by a number of personality trait variables. Model estimation results show that social dependency effects arising from similarities in attitudes and preferences are significant in explaining shared mobility service usage. Ignoring such effects may lead to erroneous estimates of the adoption and usage of future transportation technologies and mobility services.

Keywords: *spatial dependence, social interactions, attitudinal proximity, values and behavior, shared mobility service usage, latent constructs*

1. INTRODUCTION

Incorporating notions of interdependency in explaining travel patterns and locational choice behavior of decision makers has garnered much interest in the recent past (Dugundji and Walker, 2005; Bhat et al, 2016). A key differentiating factor in these studies is that they account for the nature of proximity amongst decision makers, which results in varied forms of networks over which feedback or inter-dependency effects propagate. Proximity is defined as the degree of closeness between decision makers and can be measured along different dimensions - geographic space, social space, and attitudinal space (lifestyle preferences, attitudes and values). Proximity in geographic space has traditionally accrued importance in econometric models that account for dependency amongst decision makers (Dugundji and Walker, 2005; Bhat et al, 2016), attuned to the idea that decision makers' preferences and choice behavior are shaped by dyadic exchanges between individuals in close spatial proximity of one another. However, several studies have pointed out that social influence is pervasive and a decision maker's decisions are not isolated from the influence of other decision makers in his or her social sphere (Brock and Durlauf, 2001; Arentze and Timmermans, 2008).

Recent advances in technology and the accompanying growth in social media platforms such as Facebook and Twitter have rendered spatial separation practically moot as much of social interaction occurs virtually (Hackney and Axhausen, 2006). Research in social interactions has considered associations within tight social networks such as among family members (Arentze and Timmermans, 2009) as well as wider networks extending to colleagues, friends, and virtual social media connections (Axhausen, 2008). However, there is limited knowledge of (a) the topology of such networks and their influence on transportation decisions (b) the feasibility of using global networks of decision makers in such a social space (c) methods to operationalize the strength of relationships in such networks (Hackney and Axhausen, 2006). Adding to this is the arduous and often intractable task of extracting information about social network connections from conventional travel and land-use survey data (Axhausen, 2008). As a result, research that accounts for the influence of social networks in shaping travel behavior is rather sparse. Even in the limited literature on this topic, studies have utilized associative, aggregate-level networks where decision makers are grouped by planning zone and observed socio-demographic or economic characteristics (Yang and Allenby, 2003; Dugundji and Walker, 2005).

In pursuit of a framework that can accommodate social dependency effects in studying travel behavior, this paper extends the concept of proximity-based dyadic interactions by introducing the idea of attitudes, habits and lifestyle preferences as a new dimension and measure of proximity. As opposed to the physical networks that are based on observable socio-spatial variables, *latent social networks* are introduced in this paper. In this paradigm, the inter-dependency among decision makers originates from similarity in the attitudinal space. The methodology is applied in this study to account for both interdependencies amongst decision makers in spatial-attitudinal space and dynamics of self-selection due to inherent attitudes, preferences, and habits affecting the frequency with which individuals use car-sharing and ride-sourcing mobility services.

This topic is of particular relevance as the urban transportation landscape has been significantly disrupted by the emergence of mobility-on-demand services, inspired by the concept of a sharing economy (Hannon et al, 2016). Two such services that figure prominently in this era of smart-mobility are car-sharing and ride-sourcing services. Car-sharing services, which are car-rentals by the hour or minute, afford consumers all of the benefits of automobile ownership without incurring high fixed costs of purchase, insurance, and maintenance (Shaheen et al, 2009). Ride-sourcing refers to a mobility-on-demand service that offers a lower cost alternative to taxis, provides door-to-door service, and can be hailed, monitored, and paid for using technology-based platforms (Dias et al, 2017). While some recent studies on such services have explored the role of socio-economic and built environmental factors (Coll et al, 2014; Rayle et al, 2016), there is a paucity of literature that examines inter-dependencies in attitudinal space that impact usage patterns of these mobility-on-demand services. Studies by Efthymiou et al (2013) and Dias et al (2017) have acknowledged the crucial role of underlying attitudes and lifestyle preferences and the adoption and use of such services. Latent constructs are introduced in this study to both capture effects of underlying attitudes and lifestyle preferences as well as self-selection in modeling frequency of participation in ride-sourcing/car-sharing services.

The paper uses data from the 2015 Puget Sound Regional Travel Study (PSRC, 2015) to model the monthly usage of ride-sourcing and car-sharing services for adults, which constitutes the ordinal variable of interest. The study considers two latent constructs relevant to urban travel and locational behavior: pro-environment attitude and neo-urban (active) lifestyle propensity. It should be noted that the paper's focus is only on short-term travel choices, and hence variables reflecting long-term household decisions, such as residence type and vehicle ownership, are included only as exogenous covariates for the ordinal variable of interest.

The reminder of the paper is organized as follows. The next section presents relevant literature and sets a foundation for going beyond spatial measures to define proximity in modeling travel behavior. The third section describes the dataset, while the fourth section presents the behavioral and methodological frameworks. The fifth section presents model estimation results. Discussion and concluding thoughts are offered in the sixth and final section.

2. GOING BEYOND SPATIAL MEASURES OF DEPENDENCE

The study of attitudes, perceptions, habits, and lifestyle preferences has been of interest to travel behavior researchers due to their role in shaping human activity-travel choices (Kitamura et al, 1997; Bagley and Mokhtarian, 2002). This notion is further reinforced by theories in social psychology which evaluate how such personality traits shape short-term and long-term behavior, and recognize that a decision maker's behavior often tends to conform to the social constraints and norms of the individual's cohort or reference group. Theory of Reasoned Action (Fishbein, 1980) and Theory of Planned Behavior (Ajzen, 1991) suggest that attitudes and lifestyle preferences play an important role in shaping behavior in different contexts. Subjective norms – the sum of normative beliefs due to social pressure to conform to one's reference group – also influence behavior. For example, people who perceive themselves to be pro-environmental may bicycle to work or buy a clean-fuel vehicle to align their actions with those of other pro-environmental decision makers. These three influences (attitudes, lifestyle preferences, and subjective norms), which characterize consistent patterns of behavior, have been termed as "reasoned influences". Unreasoned influences on the other hand include habits and dependencies, and trace their origins to the Theory of Repeated Behavior (Ronis et al, 1989). This theory suggests that repeated behavior is motivated more by habit than attitudes.

Unlike some of the social networks mentioned previously, a decision maker may not interact with group members in the same attitudinal space either physically or virtually (refer to example of bicycling to work). Social inter-dependency engendered through passive observation of individuals in a similar attitudinal space is a simple and powerful construct that is yet to be fully explored. It is therefore hypothesized that a decision-makers' position in attitudinal space can suppress or promote different courses of action, a behavioral phenomenon that policy makers can leverage to achieve mobility goals. Within the context of accommodating dependencies, this study adopts a spatial lag structure for the outcome variable of interest. The latent constructs reflecting attitudes, habits, and preferences are based on observed psychometric indicators and/or other variables describing observed behavior (e.g., smartphone ownership) and scores for these latent constructs are estimated using Bhat's (2015) Generalized Heterogeneous Data Model (GHDM). These latent constructs serve to introduce dependencies amongst decision makers in the attitudinal space.

In conventional spatial econometric models, the autocorrelation among decision makers is diffused via a weight matrix that is based on a spatial network measuring distances between decision makers (e.g., Paleti et al, 2013). Elements in each row of the matrix reflect the absolute spatial influence of all decision makers on a given decision maker. In this paper, the network topology is determined by both spatial (geographical) and attitudinal (non-spatial) proximities, the latter incorporating attitudes, preferences, and habits. The influence of attitudinal and spatial networks is disentangled by using coefficients for each proximity measure. This opens up the possibility for one measure counteracting the influence of another; for example, even when decision-makers are in close geographical proximity, differences in their attitudes, preferences, and habits may outweigh their spatial proximity.

3. DATA DESCRIPTION

The data for this study is derived from the Puget Sound Regional Travel Study that involved survey data collection in 2014 and 2015 covering a five-county area in the State of Washington. In addition to collecting information about socio-economic, demographic, and activity-travel characteristics, the survey asked respondents to provide information about attitudes, preferences, and technology (e.g., smartphone) ownership and usage. Data about residential location choice preferences, and membership and usage of shared mobility services such as ride-hailing, bike-share, and car-share services, was collected through the survey. All relevant variables used in this study were extracted from the 2015 edition of the survey data set, except for the variables that capture the frequency of use of technology platforms (smartphone apps and websites) for obtaining travel-related information. These two variables are available in the 2014 edition of the survey; these variables are imputed into the 2015 data set based on ordered probit models of technology use estimated on the 2014 data. The resulting imputed variables are ordinal variables that measure the frequency of use of technology platforms for travel-related information. The categories of the ordinal variables, based on measures of usage over the past 30 day period, are: never, less than one day per week, one day per week, and two or more days per week.

The analysis is limited to adults (age 18 years or above). The dependent variable of interest is the frequency of using ride-sourcing services (e.g., Uber and Lyft) and/or car-sharing services (e.g., ZipCar and car2go) in the past 30 days. Information on this variable is derived from ordinal indicators measuring level of usage as reported by the respondents. The seven-level ordinal scale includes the following:

- Never
- I do this, but not in the past 30 days
- 1-3 times in the past 30 days
- 1 day per week
- 2-4 days per week
- 5 days per week
- 6-7 days per week

The two disruptive mobility services are considered together in this study because both are technology-enabled, and involve the use of vehicles not owned by the traveler. To account for very small sample sizes in some categories, and for computational tractability, a more aggregate three-point ordinal scale was used to represent the level of usage:

- Never
- Occasionally, but not in the past 30 days
- Used service in past 30 days with any frequency

The final cleaned and filtered sample used for analysis and model estimation included 2170 adults. A majority of the individuals in the analysis sample are in the middle age groups. There are more females than males, and full-time employed individuals constitute nearly one-half of the sample. About 36 percent of the sample is unemployed. Only about six percent of the sample reported being a student, a similar percent reported not having a driver's license, and about 70 percent of the sample reported owning a smartphone. About 12 percent of the sample resides in households with no vehicles, about 30 percent of the sample report living in high-density census blocks of 5000 or more households per square mile. Nearly 20 percent of the sample reside in single-person households, and an almost equal percent reside in nuclear family households with children. The income distribution shows that 34 percent of individuals reside in households that make over $100,000 per year. Only 10 percent of the sample has membership in car- or bike-share services. An examination of the dependent variable of interest shows that 81 percent of the sample has never used car-share or ride-sourcing services in the past 30 days. This is consistent with the notion that shared mobility services are relatively new entrants in the transportation landscape. A detailed tabulation of descriptive sample characteristics is not provided in the interest of brevity, but may be found in Vinayak et al (2017).

4. BEHAVIORAL AND METHODOLOGICAL FRAMEWORKS

This section offers a detailed description of the behavioral and methodological frameworks adopted in this study.

4.1 Linking Latent Constructs With Usage Patterns – A Behavioral Framework

The behavioral framework adopted in this study is shown in Figure 1. Latent constructs that describe an individual's innate attitudes and lifestyle preferences are linked to the proclivity to adopt and use shared mobility services in this framework. Latent attitudinal constructs are modeled as functions of exogenous variables and appear in the data set as indicator variables that represent observed travel and locational choice behavior as well as responses to attitudinal questions. Instead of explicitly modeling the impacts of these latent constructs on shared mobility service usage, the latent constructs are used to induce dependency effects over a latent social network of individuals who are proximally located in attitudinal space over and above the dependency effects attributed to spatial proximity. Latent factors considered in this study include an individual's "pro-environmental attitude" and "neo-urban lifestyle propensity", both of which have surfaced repeatedly in the literature as determinants of activity-travel choices, especially in the context of shared mobility service usage (Lavieri et al, 2017; Astroza et al, 2017).

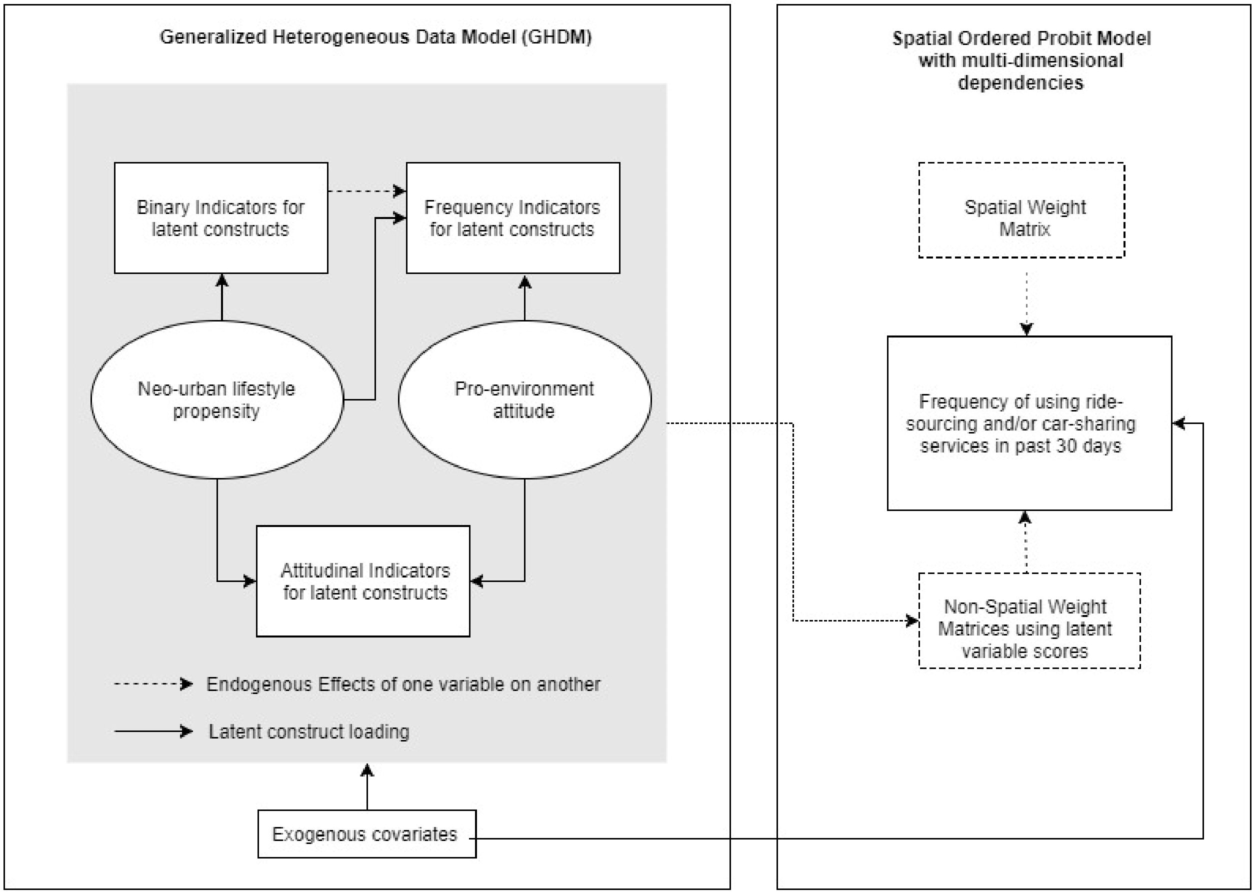

Figure 1. Overview of Behavioral Framework

A pro-environmental attitude has been found to be significantly associated with shared mobility use (e.g., Efthymiou et al, 2013; Burkhardt and Millard-Ball, 2006). It has been shown in these studies that pro-environmental individuals eschew use of personal vehicles in favor of the use of transit and non-motorized modes and exhibit a higher affinity towards use of ride-sourcing and car-sharing services. In this study, four ordinal variables in the data set are considered representative of a pro-environmental attitude:

- Importance of residing close to transit (measured on a five-point scale: very unimportant to very important)
- Importance of residing in a walkable neighborhood with access to local activities located nearby (measured on a five-point scale: very unimportant to very important)
- Frequency of bicycling episodes (more than 15 minutes) in past 30 days (measured on a four point scale: never, I do – but not in past 30 days, more than once in past 30 days – but at most one day per week, and two or more days per week)
- Frequency of walking episodes (more than 15 minutes) in past 30 days (measured on the same four-point scale as frequency of bicycling episodes)

The neo-urban lifestyle propensity is comprised of three unique features – use of technology to access travel-related information, proclivity for shared-space and collaborative ownership (i.e., proclivity to participate in the shared economy), and level of importance attached to residing in locations close to work and social-recreational activities. It has been shown in previous studies that these three attitudinal traits are significantly associated with the use of car-share and ride-sourcing services (Astroza et al, 2017; Montgomery, 2015). In this study three ordinal variables and two binary outcomes are tested as indicators of a neo-urban lifestyle propensity:

- Frequency of using technology-based platforms (smartphone apps and/or websites) for travel information in past 30 days (measured on same four-point scale as frequency of walking and bicycling episodes)
- Smartphone ownership (binary indicator)
- Level of interest in participating in an autonomous vehicle car-share system (measured on a five-point scale: not at all interested to very interested)
- Importance of residing in a home location close to highways or major roads (measured on a five point scale: very unimportant to very important)
- Importance of living within a 30-minute work commute (measured on a five scale: very unimportant to very important)

Table 1 presents a summary of the indicator variables for the analysis sample. Being close to highways and major roads is generally considered less important than being within a 30-minute work commute and having a walkable neighborhood with local activities nearby.

TABLE 1. Descriptive statistics of indicator variables

Attitudinal (Ordinal) Indicator Variables					
	Response Distribution				
Importance of factor in choosing home location	Very Unimportant 1	Unimportant 2	Neutral 3	Important 4	Very Important 5
Close to major roads/highways	14.8%	16.0%	22.2%	34.6%	12.4%
Being within 30-minute commute to work	11.0%	6.1%	17.7%	20.4%	44.8%
Being close to public transit	15.4%	10.4%	17.8%	25.3%	31.1%
Having a walkable neighborhood and being near local activities	5.3%	6.7%	10.3%	33.2%	44.4%
	Response Distribution				
Level of interest in use of…	Not at all interested	Somewhat uninterested	Neutral	Somewhat interested	Very Interested
Autonomous car-share system for daily travel	55.4%	6.7%	11.9%	14.0%	11.7%

Frequency (Ordinal) Indicator Variables				
	Response Distribution			
Frequency of participating in…	Never	I do, but not in the past 30 days	More than once in past 30 days but at most 1 day/week	Two or more days/week
Bicycling (15 min or more)	62.7%	20.7%	8.4%	8.2%
Walking (15 min or more)	8.3%	6.0%	18.7%	66.9%

Frequency of…	Smartphone ownership and app use for travel info	*Frequency of:*	Technology-based platforms for travel info
Don't own smartphone	30.0%	Never	31.1%
Own smartphone but never use apps for travel info	21.8%	Less than one day per week	33.2%
Own smartphone and use apps less than one day per week for travel info	18.9%	One day per week	12.9%
Own smartphone and use apps one or more days per week for travel info	29.4%	Two or more times per week	22.8%

A majority of the sample is not at all interested in using an autonomous car-share system for daily travel. The frequency of walking is substantially larger than the frequency of bicycling. About 22 percent own a smartphone, but never use apps for travel information. About 30 percent own a smartphone and use apps one or more days per week for travel information. About 31 percent of the sample never uses technology platforms for travel information. On the other hand, 23 percent do so two or more times per week. The statistics in the table show that there is considerable heterogeneity in the population with respect to residential location preferences, interest in autonomous car-share adoption, and use of technology platforms for travel information.

The modeling framework consists of two primary components, namely, the Generalized Heterogeneous Data Model (GHDM) and the spatially lagged ordinal response model with a composite weight matrix that includes both spatial and aspatial (attitudinal) components. Within the GHDM, there are two submodels – a latent structural equation model (SEM) and a latent measurement equation model (MEM). In the latent SEM, the latent psychological constructs are represented as linear functions of exogenous variables with the usual stochastic error terms. In the latent MEM component, psychometric indicators along with observed travel behavior indicators are posited as functions of latent constructs, exogenous variables, and other endogenous outcomes. The SEM and MEM sub-models are estimated jointly in a simultaneous equations modeling framework. However, because the emphasis of this study is on the spatially lagged ordinal response model with a composite weight matrix, the presentation of the methodology in the next section focuses on the second component of the model system. Details about the GHDM formulation to model the latent constructs can be found in Bhat (2015). The GHDM is estimated using the Maximum Approximate Composite Marginal Likelihood (MACML) approach (Bhat, 2011).

4.2 Capturing Dependency Effects Using a Spatial Lag Structure

This section describes the approach to model the ordinal variable of interest (frequency of car-sharing and/or ride-sourcing in the past 30 days) with spatial-attitudinal dependency effects. The ordinal variable has three levels corresponding to usage of shared mobility services: never, occasionally but not in the past 30 days, and one or more times in the past 30 days.

The use of a spatial lag structure allows choice behavior of a decision maker to be influenced by that of peers in the geographic-attitudinal space. While proximity in geographic space is derived using spatial distances between residence locations of decision makers, the proximity in attitudinal space is based on a latent social network defined by similarities in attitudes and lifestyle preferences. These are captured by the two latent constructs considered in the study: pro-environment attitude and neo-urban lifestyle propensity. The dependency effects due to each dimension of proximity are disentangled using separate coefficients for each proximity measure.

In the following discussion, consider a sample of Q decision makers denoted by index $q(1,2,3...,Q)$ and L latent variables denoted by index l (L=2 in this study). Let each latent variable for individual q be represented by z_{ql}. Each of these latent variables is written as a function of observed covariates and a stochastic error component within the SEM submodel of the GHDM. For later use, collect all of the constructs for latent variable l across all individuals q in the vector $\boldsymbol{z}_l = (z_{l1}, z_{l2}, ..., z_{lQ})'$. Also, let the expected value of this vector, as obtained from the GHDM, be $\hat{\boldsymbol{z}}_l$.

The ordinal variable of interest for decision maker q, in the spatial lag structure, is specified in terms of exogenous covariates as follows,

$$\tilde{y}_q = \rho \sum_{q'=1}^{Q} w_{qq'} \tilde{y}_{q'} + \tilde{\boldsymbol{\gamma}}' \boldsymbol{x_q} + \xi_q \text{ , } y_q = k \text{ if } \psi_{q,k\text{-}1} < \tilde{y}_q < \psi_{q,k} \tag{1}$$

where $\tilde{y}_q$ is the underlying continuous latent response variable whose partitioning relates to the K levels of the ordinal variable, and $\tilde{\boldsymbol{\gamma}}$ $(A\times 1)$ is the vector of coefficients associated with the $\boldsymbol{x_q}$ $(A\times 1)$ vector of exogenous covariates (excluding the constant). Let the idiosyncratic error term ξ_q be standard normally distributed and independently and identically distributed across decision makers. Let $w_{qq'}$ be the (q,q') element of the row-normalized multi-dimensional weight matrix $\boldsymbol{W}$ $(Q\times Q)$ with zeros on the diagonal ($w_{qq} = 0$, $\sum_{q\neq q'}^{Q} w_{qq'} = 1$) and ρ $(0<\rho<1)$ be the auto-regressive parameter. In vector notation, the consolidated formulation for all individuals Q is given as,

$$\tilde{\boldsymbol{y}} = \rho \boldsymbol{W} \tilde{\boldsymbol{y}} + \boldsymbol{x} \tilde{\boldsymbol{\gamma}} + \boldsymbol{\xi} \tag{2}$$

where $\tilde{\boldsymbol{y}} = (\tilde{y}_1, \tilde{y}_2, ..., \tilde{y}_Q)'$ and $\boldsymbol{\xi} = (\xi_1, \xi_2, ..., \xi_Q)'$ are $(Q\times 1)$ vectors, $\boldsymbol{x}$ is $(Q\times A)$ matrix of exogenous variables for individuals. Through a simple matrix operation, the equation can now be rewritten as:

$$\tilde{\boldsymbol{y}} = \boldsymbol{T}\boldsymbol{x}\tilde{\boldsymbol{\gamma}} + \boldsymbol{T}\boldsymbol{\xi} \tag{3}$$

$$\boldsymbol{T} = (\boldsymbol{I_Q} - \rho \boldsymbol{W})^{-1} \tag{4}$$

where $\boldsymbol{I_Q}$ is an identity matrix of size Q. The vector $\tilde{\boldsymbol{y}}$ is multivariate normally distributed with mean $\boldsymbol{T}\boldsymbol{x}\tilde{\boldsymbol{\gamma}}$ and covariance matrix $\boldsymbol{TT}'$, i.e., $\tilde{\boldsymbol{y}} \sim \boldsymbol{MVN_Q}(\boldsymbol{T}\boldsymbol{x}\tilde{\boldsymbol{\gamma}}, \boldsymbol{TT}')$.

The crux of this paper lies in the formulation of the composite weight matrix $\boldsymbol{W}$, which engenders the interdependencies amongst decision makers in geographic and attitudinal space. The composite weight matrix is a combination of spatial and non-spatial (one corresponding to each latent construct) weight matrices. Unlike previous formulations (e.g., Yang and Allenby, 2003), the number of constituent weight matrices does not explode with an increasing number of non-spatial measures of proximity. Instead the non-spatial proximity (in attitudinal space) is parsimoniously expressed using a reduced number of latent variable - distance matrices. The composite weight matrix $\boldsymbol{W}$ $(Q\times Q)$ is specified as follows,

$$\boldsymbol{W} = \boldsymbol{exp}(-(\boldsymbol{D_{spatial}} + \sum_{\boldsymbol{l}=1}^{\boldsymbol{L}} \kappa_l \boldsymbol{D^l_{non-spatial}})) \tag{5}$$

where, $\boldsymbol{D_{spatial}}$ is a $(Q\times Q)$ spatial distance matrix that is derived using latitude-longitude coordinates of decision makers' residential locations. $\boldsymbol{D^l_{non-spatial}}$ is the $(Q\times Q)$ non-spatial distance matrix, based on attitudinal proximity on latent variable l $(1,2,...,L)$. Further, $\{\kappa_1, \kappa_2, ..., \kappa_L\}$ are coefficients associated with the non-spatial proximity measures derived from each of the L latent variables. The element-by-element exponentiation operator allows for negative values for kappa while still ensuring non-negativity of the final weights. The coefficient associated with spatial distance is fixed to unity to ensure econometric identification.

The non-spatial distance matrix $\boldsymbol{D^l_{non-spatial}}$, associated with latent variable l, is populated using a $(Q\times Q)$ matrix $\hat{\boldsymbol{Z}}_{\boldsymbol{l}}$ that is expressed as the Kronecker product of $\hat{\boldsymbol{z}}_{\boldsymbol{l}}$ $(Q\times 1)$ vector of predicted values for latent variable l and a $(1\times Q)$ row vector of ones. Due to the nondirectionality of differences in latent lifestyles and preferences across decision makers, the absolute difference of $\hat{\mathbf{Z}}_l$ with its transpose $\left(\hat{\mathbf{Z}}_l\right)'$ is taken. This results in a $(Q\times Q)$ distance matrix of attitudinal proximity on latent variable l given by,

$$\hat{\mathbf{Z}}_l = \hat{\boldsymbol{z}}_l \otimes ones(1,Q) = \begin{bmatrix} \hat{z}_{1,l} \\ \hat{z}_{2,l} \\ \dots \\ \hat{z}_{Q,l} \end{bmatrix} \otimes \begin{bmatrix} 1 & 1 & \dots & 1 \end{bmatrix}_{1\times Q} \tag{6}$$

$$\boldsymbol{D}^{l}_{non-spatial} = |\hat{\mathbf{Z}}_l - (\hat{\mathbf{Z}}_l)'| = \begin{bmatrix} 0 & |\hat{\boldsymbol{z}}_{1,l} - \hat{\boldsymbol{z}}_{2,l}| & \dots & |\hat{\boldsymbol{z}}_{1,l} - \hat{\boldsymbol{z}}_{Q,l}| \\ |\hat{\boldsymbol{z}}_{2,l} - \hat{\boldsymbol{z}}_{1,l}| & 0 & \dots & |\hat{\boldsymbol{z}}_{2,l} - \hat{\boldsymbol{z}}_{Q,l}| \\ \dots & \dots & \dots & \dots \\ |\hat{\boldsymbol{z}}_{Q,l} - \hat{\boldsymbol{z}}_{1,l}| & |\hat{\boldsymbol{z}}_{Q,l} - \hat{\boldsymbol{z}}_{2,l}| & \dots & 0 \end{bmatrix}_{Q\times Q} \tag{7}$$

An important note here is that the non-spatial proximity measures among decision agents, as constructed above, are based on the expected values of the latent constructs as opposed to their actual values. The main reason for this formulation is that the sample is but a random fraction of the population of interest. It is impossible to represent every individual in spatial or social space, and therefore more appropriate to consider a sampled neighbor in spatial or social space as representative of many others in the population who may be in that space. It may then be intrinsically more appropriate to consider the *expected value* of a sampled neighbor's latent construct (representing the larger set of individuals in the population with the same observed characteristics that impact the latent variable of the sampled neighbor), and examine the distance of this expected value from the expected value of the sampled individual in question.

The spatial distances matrix ($\boldsymbol{D}_{spatial}$) and the non-spatial distance matrices ($\boldsymbol{D}^{l}_{non-spatial}$) are normalized (divided by the maximum value) before they enter Equation (6) to adjust for scale differences. Prior to feeding the composite weight matrix $\boldsymbol{W}$ into the SORP model (Equation 2), the diagonal elements of $\boldsymbol{W}$ are set to zero and $\boldsymbol{W}$ is row-normalized to ensure that each decision maker gets the same net influence from all other decision makers.

The parameters to be estimated in the ordered probit model with spatial and non-spatial dependencies are the vector of exogenous coefficients $\tilde{\boldsymbol{\gamma}}$, the auto-correlation parameter ρ, $(M-1)$ thresholds of the ordinal variable ($\psi_0 = -\infty, \psi_K = \infty, -\infty < \psi_1 < \psi_2 ... < \psi_{K-1} < \infty$), and κ $\{\kappa_1, \kappa_2, ..., \kappa_L\}$ coefficients associated with the non-spatial weight matrices. The likelihood function $L(\boldsymbol{\theta})$ for the model takes the following form,

$$L(\boldsymbol{\theta}) = P(\boldsymbol{y} = \boldsymbol{m}) = \int_{D_{\tilde{y}}} \boldsymbol{F_Q}(\tilde{\boldsymbol{y}} \,|\, \boldsymbol{Tx}\tilde{\boldsymbol{\gamma}}, \boldsymbol{TT}')d\tilde{\boldsymbol{y}} \tag{8}$$

where $\boldsymbol{\theta} = (\tilde{\boldsymbol{\gamma}}', \rho, \kappa_1, \kappa_2, ..., \kappa_L, \psi_1, \psi_2, ..., \psi_{K-1})'$ is the $((A+L+K)\times 1)$ vector of coefficients to be estimated, $\boldsymbol{y} = \left(y_1, y_2, ..., y_Q\right)$, $\boldsymbol{m} = \left(m_1, m_2, ..., m_Q\right)$ is the $(Q\times 1)$ vector of actual observed level of frequency of using car-sharing and/or ride-sourcing. $D_{\tilde{y}}$ is the domain of integration defined as $D_{\tilde{y}} = \left\{\tilde{y} : \psi_{q,m-1} < \tilde{y}_q < \psi_{q,m},\ \forall q = 1,2,...,Q\right\}$. $\boldsymbol{F_Q}(.)$ is the Q-variate normal cumulative function with mean $\boldsymbol{Tx}\tilde{\boldsymbol{\gamma}}$ and correlation matrix $\boldsymbol{TT}'$. The autoregressive parameter ρ is reparametrized as

$\rho = \dfrac{exp(\breve{\rho})}{1+exp(\breve{\rho})}$ to ensure that $0 < \rho < 1$ and the likelihood function is maximized with respect to $\breve{\rho}$. The true value of ρ can be easily extracted after the estimation process. The likelihood function is maximized using a pair-wise composite marginal likelihood (CML) approach (Bhat, 2011). Dependency effects dilute very quickly as distance between observations increases (Castro et al, 2013). Based on statistical tests discussed in Bhat (2011), a distance threshold of eight miles is adopted and only those pairs of observations falling within this distance band are included in the CML function.

5. MODEL ESTIMATION RESULTS

This section presents a detailed discussion of the model estimation results of the GHDM and Spatial Ordered Response Probit (SORP) model components with various forms of dependency effects among decision-makers. The final model specification was adopted, after testing an extensive number of alternative specifications, based on a combination of behavioral interpretation and statistical significance. In the following sections, only results for the SEM submodel of the GHDM (which determines the latent constructs) and the SORP model component are presented. The MEM submodel is not of primary importance in this paper; it simply serves as the vehicle to estimate the SEM submodel by establishing correspondence between latent constructs and their observed indicators. Detailed results of the MEM submodel may be found in Vinayak et al (2017).

5.1 Structural Equation Model (SEM) Component of GHDM

Table 2 presents estimation results for the SEM component of the GHDM. In general, results are behaviorally intuitive and consistent with expectations. Younger individuals are likely to be more pro-environmental and favor a neo-urban lifestyle. Females exhibit a greater sensitivity to the environment, a finding consistent with previous research (Kalof et al, 2002). Income is strongly related to pro-environmental attitudes, with individuals in lower income households exhibiting greater levels of the pro-environmental attitude. Individuals with a college education are likely to be pro-environmental and favor active neo-urban lifestyles, consistent with the notion that they are likely to have greater awareness of the ill-effects of pollution. Households with children are more likely to reside in suburban locations in larger homes; consistent with such a lifestyle, individuals in these households express lower levels of the pro-environmental attitude or preference for a neo-urban lifestyle. An interesting finding is that the correlation between error terms is insignificant. The model specification may have captured all key effects, or it is possible that positive and negative correlations due to unobserved effects canceled out.

5.2 Spatial Ordered Response Probit (SORP) Model with Dependency Effects

Table 3 presents estimation results for the SORP model with spatial and non-spatial (attitudinal) dependencies. The dependent variable is the frequency of using shared mobility services.

For comparison purposes, models with no dependency effects and only spatial dependency effects (autocorrelation) are also presented alongside the SORP model that incorporates multi-dimensional spatial and non-spatial dependencies. In general, frequent users of shared mobility services are younger individuals, more educated, employed full-time, and reside in higher income households. All of these indications are consistent with findings reported elsewhere in the literature (e.g., Smith, 2016; Dias et al, 2017). Those who own smartphones are more likely to use shared mobility services; this is presumably because the use of shared mobility services often requires the ownership of a smartphone. Female smartphone owners who use apps fairly regularly for travel information are less likely to use shared mobility services, possibly due to safety considerations and the consistent finding reported in the literature that females carry a greater burden of chauffeuring and household maintenance activities, thus engendering greater levels of trip chaining and joint travel (Garikapati et al, 2014). Such travel patterns are not as conducive to shared mobility service usage. Higher levels of vehicle ownership are associated with lower levels of shared mobility service use frequency, a finding that is consistent with expectations and prior literature (Coll et al, 2014).

Table 2. Estimation Results for Structural Equation Model of GHDM

Structural Equation Component	**Pro-environment attitude**		**Neo-urban lifestyle propensity**	
Variable	**Coefficient**	**t-stat**	**Coefficient**	**t-stat**
Age (base: 55 + years old)				
18 to 24 years old	0.565	3.12	1.648	4.22
25 to 34 years old	0.374	4.31	1.396	4.55
35 to 44 years old	0.423	4.35	1.208	4.65
45 to 54 years old	0.183	1.99	--	--
Female (base: male)	0.137	2.13	--	--
Education (base: lower than Bachelor's)				
Bachelor's Degree	0.432	5.64	0.489	4.75
Graduate Degree	0.678	7.84	0.500	4.65
Income (base: $75,000 or more per year)				
Less than $24,999 per year	0.552	4.94	--	--
$25,000 - $49,999 per year	0.110	1.34	--	--
$50,000 - $74,999 per year	0.104	1.27	--	--
Employment Status (base: Unemployed)				
Full-time, part-time or self-employed	0.164	2.33	1.032	4.73
Household Structure (base: no kids)				
Atleast 1 kid (0-17 years)	-0.325	-3.79	-0.306	-1.78
Correlation between latent variables	--			

What is particularly noteworthy is that the model coefficients differ in magnitude among the model forms. This suggests that the use of models that do not account for dependencies may offer erroneous forecasts and estimates of policy impacts. The spatial correlation term, ρ, is statistically significant in both models that account for spatial dependencies; thus accounting for proximity or social dependency in the attitudinal space does not necessarily eliminate the significant effects of spatial proximity. In addition, however, parameters representing social dependency arising from proximity in the attitudinal space are also statistically significant for both attitudinal constructs considered in this paper. They are positive in value, suggesting that diffusion effects are at play. As more people use shared mobility services, the more visible they become to the rest of the population – both from a spatial perspective and a social (attitudinal and lifestyle) perspective. Finally, the adjusted composite likelihood ratio test (ADCLRT) was used to compare model fit, and it was found that the SORP that accounted for multi-dimensional dependencies performed significantly better than other model forms. The composite log-likelihood (CLL) function values at convergence are respectively -837319, -680959, and -637788 for the aspatial ORP, SORP with only spatial dependency, and SORP with multi-dimensional dependency. The ADCLRT computations yield χ^2 statistics that are statistically significant at any level of confidence, demonstrating the importance of accounting for multi-dimensional dependency effects in activity-travel choice models.

Table 3. SORP Model with Spatial and Non-Spatial Dependencies

Exogenous effects on frequency of using ride-sourcing and/or car-sharing in past 30 days	Aspatial ORP		SORP with Spatial Dependencies Only		SORP with Spatial & Non-Spatial Dependencies	
	Coefficient	t-stat	Coefficient	t-stat	Coefficient	t-stat
Age (base: 45 or more years)						
18 to 24 years old	0.881	4.49	0.906	10.97	0.598	5.26
25 to 34 years old	0.661	6.90	0.777	16.11	0.492	5.04
35 to 44 years old	0.527	5.41	0.573	12.05	0.336	3.77
Work Status						
Full-time, part-time or self-employed *(base: unemployed)*	0.381	3.95	0.432	9.02	0.227	3.37
Student *(base: not a student)*	0.253	1.93	0.172	2.94	0.254	12.95
Income (base: above $100,000)						
Below $25,000	-0.684	-4.57	-0.445	-6.57	-0.741	-19.34
$25,000 - $49,999	-0.581	-5.10	-0.513	-8.49	-0.736	-23.31
$50,000 - $74,999	-0.366	-3.26	-0.194	-3.30	-0.385	-19.21
$75,000 - $99,999	-0.397	-3.51	-0.156	-2.59	-0.318	-17.27
Educational attainment (base: less than a bachelor's degree)						
Bachelor's degree	0.386	3.78	0.217	4.10	0.184	4.87
Graduate degree	0.430	4.00	0.249	4.66	0.182	4.13
Smart-phone ownership and frequency of usage for travel information in past 30 days (base: don't own a smart-phone)						
Own smart-phone but never use apps	0.881	6.42	0.923	14.25	0.959	22.52
Own smart-phone and use apps less than once a week	0.800	5.77	0.799	12.22	0.834	22.81
Own smart-phone and use apps once or more a week	1.080	7.62	1.079	16.49	1.116	23.31
Own smart-phone and use apps once or more a week x Female	-0.249	-2.28	-0.222	-4.27	-0.263	-14.16
Residential Location Density (base: Low Density)						
High Density	0.694	7.64	0.246	5.83	0.497	25.40
High Density x Presence of atleast one kid	-0.416	-2.15	-0.293	-2.95	-0.300	-11.25
Vehicle Ownership and Residence Type (base: no vehicles)						
One Vehicle and single-family residence	-0.505	-3.51	-0.344	-4.85	-0.606	-21.45
Two or more Vehicles and single-family residence	-1.207	-8.32	-0.892	-12.58	-1.259	-26.21
One Vehicle and multi-family residence	-0.653	-5.43	-0.564	-10.57	-0.648	-23.19
Two or more Vehicles and multi-family residence	-0.608	-3.79	-0.383	-5.41	-0.597	-20.69
ρ	--	--	0.562	2.24	0.895	2.02
κ_1 (pro-environment attitude)	--	--	--	--	0.883	2.79
κ_2 (neo-urban lifestyle propensity)	--	--	--	--	1.151	2.53

6. DISCUSSION AND CONCLUSIONS

Individuals interact with one another as an inevitable part of living in a society. People observe what others do, interact and exchange information with others, and modify their own behaviors, choices, attitudes, and goals in response to societal forces. Yet, many travel models continue to ignore the forces of inter-dependency when simulating activity-travel choices. Models (largely in the research domain) that recognize inter-dependency are often limited to accounting for intra-household interactions among family members. Models that purport to capture influences beyond the immediate confines of the household do so through spatial dependency effects that are purely based on measures of geographic proximity. People may purchase environmentally friendly vehicles, bicycle and walk, use transit, or let their children walk to school in response to observing what their neighbors do and interacting with them.

However, in an era of social media platforms and ubiquitous connectivity, inter-dependencies may no longer be solely influenced by geographic proximity. Rather, the strength of association among individuals may be influenced by attitudes, values, preferences, and perceptions. Those with similar attitudes and lifestyle preferences may interact more closely (for example, in online communities and forums), thus enhancing social dependency effects among such individuals who share comparable perspectives.

This paper makes a fundamental contribution to the literature by proposing an econometric methodology that is capable of simultaneously accounting for both spatial and non-spatial (attitudinal) dependency effects. The model system takes the form of a simultaneous equations model system with latent constructs that describe individual attitudes and lifestyle preferences as a function of measured indicators in survey data. The proximity among individuals with respect to the latent constructs is explicitly incorporated (along with spatial measures of separation) into the weight matrix that captures the strength of association across observations. The formulation is able to disentangle the strength of the inter-dependency due to attitudinal proximity from that due to spatial proximity.

The model system is applied to the study of the frequency of use of shared mobility services, including car-sharing and ride-sourcing services. Two latent constructs, representing pro-environmental attitude and preference for a neo-urban lifestyle, are used to account for non-spatial dependency effects. A spatially ordered response model (SORP) is estimated within a larger Generalized Heterogeneous Data Model (GHDM) framework to examine the dependency effects. It is found that both spatial and non-spatial (attitudinal) dependency effects are significant in explaining the use of emerging shared mobility services and that both of these effects are comparable in magnitude. The model that accounted for both sources of dependency offered statistically better goodness-of-fit than models that ignored one or both sources.

The model system shows that diffusion effects are at play, not just based on distance but also based on non-spatial attitudinal and lifestyle variables. Such models can help in developing estimates of market adoption of emerging transportation technologies as they capture the diffusion effects engendered by multiple sources. Policy strategies aimed at enhancing shared mobility service usage can be better informed via models that capture various inter-dependency effects. Agencies interested in seeing greater adoption of these services could identify virtual groups and forums that may be targeted for information campaigns, incentives and rebates, and seeking assistance in spreading the word. Through such mechanisms, agencies may be able to realize significant change in behavior in response to various strategies by leveraging the power of diffusion effects that influence people's activity-travel choice behaviors.

ACKNOWLEDGEMENTS
This research was partially supported by the Center for Teaching Old Models New Tricks (TOMNET) as well as the Data-Supported Transportation Operations and Planning (D-STOP) Center, both of which are Tier 1 University Transportation Centers sponsored by the US Department of Transportation.

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